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From Wall Street to Main Street – How Banks Can Offload Their Asset Risk onto Retail Investors**

Abstract

We examine the motives of financial institutions to issue retail structured products as a funding source and as a tool for risk management. For this purpose, we construct a Merton-type model with taxes and bankruptcy costs. High-risk issuers can increase their firm value and stability given they keep the leverage ratio fixed. If issuers optimally adjust their capital structure, then they add retail structured products to the financing mix when their assets are risky and the correlation to the underlying asset is positive. Principal protected notes make default more likely, whereby discount notes decrease default risk.

JEL Classification: G01, G13, G21, G32.

Keywords: Financial Institutions; Hedging; Retail Structured Products; Risk Management.

1 Introduction

When systemic risks are a matter of concern and banks are considered to be too big to fail, hedging between banks does little to help restore trust. Risks are passed on from one financial institution to another but can still spread within the financial sector. Hence, there

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** The authors gratefully acknowledge the financial support of the German Research Foundation (Deutsche Forschungsgemeinschaft, DFG) (research grant KO 4334/2-1). Part of this work was done while Marc Crummenerl was a visiting scholar at the NYU Stern School of Business, supported by the German Academic Exchange Service (Deutscher Akademischer Austauschdienst, DAAD).

is a need to transfer risks outside the financial system and for products capable of doing so. Retail structured products could be a suitable vehicle for this kind of risk transfer.

Retail structured products, which are often advertised under the generic term *certificate*, are part of the unsecured subordinated debt of a financial institution. Their repayment is tied to the performance of an arbitrary underlying asset (mostly equities, but these can also be commodities and interest rates). Thus, with the notable exception of the issuer's bankruptcy, the repayment is not linked to the issuer's own financial performance. In contrast to mutual funds, whose assets are separated from the assets of the managing firm, the issuer's use of the proceeds is not restricted or regulated, i.e., the funds can be used for purposes other than hedging.

These derivative products, which are tailored to the needs of retail investors, have themselves come under scrutiny in the aftermath of the financial crisis. Retail investors incurred significant losses from products issued by the defaulted investment bank Lehman Brothers. Subsequently, these products and their regulation have become subject to controversial debate among policymakers and industry professionals, which mainly focuses on transparency and risks from the retail investors' point of view. However, the debate does not include the more important point of the impact on the risk choice and stability of the issuing financial institution, which is our focus in this paper.

The literature on retail structured products so far has considered these products primarily as a source for profits for the issuing banks, since the products are sold at a price well above the value from stand-alone duplication (see, e.g., Wilkens, Erner, and Röder (2003) and Stoimenov and Wilkens (2005)). We are adding two novel themes to this literature. Each retail structured product can be decomposed into a risk-free component and a derivative component. The first component is a valuable source of funds for the issuer's core business. We believe that the second component is an innovative tool for risk management.

For all standard product types, the first component is strictly positive, such that retail structured products generate a cash surplus[1]. We argue that the issuing financial institution uses the cash surplus to fund its ordinary business, for example, by granting loans, instead of purchasing risk-free government bonds. Thus, if the asset portfolio is illiquid or subject to price shocks, then the investors in retail structured products are exposed to the business risk of the issuer. The default by the prominent issuer Lehman Brothers provides anecdotal evidence for this risk exposure.

1 The German Derivative Association, which represents the issuing institutions in Germany, estimates a market size of €90.2 bn (as of end 2013). This corresponds to 1.2% of total bank liabilities and 24.4% of aggregated bank equity in Germany. For some banks, the market value of issued retail structured products already exceeds the volume of equity financing.

The first component links the payoff of retail structured products to the financial performance of the issuer; the derivative component creates an exposure to the underlying security. On the one hand, the issuer can effectively transfer a risk exposure to the retail investor, i.e., outside the banking system. On the other hand, the retail investor explicitly wants to have this exposure to the underlying asset, which is usually in the focus of the advertisements of these products. The bundle of the derivative component with a risk-free component ensures that there is no future cash flow from the retail investor to the issuer, i.e., from the issuer's perspective there are no settlement costs and no counterparty risk. Our main objective in this paper is to evaluate the conditions under which the issuers can benefit from retail structured products as a risk management tool.

To meet our main objective, we incorporate retail structured products in a simple Merton-type model. We focus on the two most prominent types of claims, principal-protected notes and discount notes. The payoff of principle-protected notes is convex in the value of the underlying asset, while the payoff of discount notes is concave. We assert that these two claims represent the class of claims with convex or concave payoffs, respectively. We use the option pricing theory developed by Black and Scholes (1973) and Merton (1973) for the consistent valuation of the retail structured products as well as all other claims in the Merton model.

According to the seminal work of Modigliani and Miller (1958), the value of the issuer is invariant to its capital structure. There is no optimal capital structure in a world without frictions. Similarly, there is no additional value to be created by risk management. Hence, there is no rationale for the existence of retail structured products in a frictionless world. As a consequence of this central result of Modigliani and Miller (1958), we have to consider market frictions to explain the issuer's capital structure choice. Hence, we incorporate the classical trade-off between tax benefits of debt and bankruptcy cost.

We find that when the assets are highly correlated with the underlying security, retail structured products increase the value of the issuer. We show that compared to the case of straight debt financing, a high-risk issuer can always improve its value and simultaneously lower the default probability for any given target leverage ratio. The opposite is true for a low-risk issuer, whose assets are uncorrelated to the underlying security.

Nevertheless, the issuer is subject to risk-shifting and has an incentive to optimally adjust its leverage and asset risk weight. Even when accounting for these optimal decisions, risky issuers prefer to optimally add retail structured products to the financing mix. Thereby, issuers with high asset risk increase the probability of default when issuing principal-protected notes, but reduce it by issuing discount notes. The results also hold when the issuer can optimally design the retail structured products.

The paper is organized as follows. In Section 2 we survey the relevant literature. In Section 3 we introduce the model and describe the valuation of all relevant claims. In Section 4 we analyze the issuer value for a given asset composition and leverage, and evaluate the issuer's optimal financing choice in Section 5. In Section 6, we analyze the risk-taking incentives of the issuer. In Section 7 we derive the optimal design of retail structured products. We also discuss further product types and product complexity. Section 8 concludes the paper.

2 Literature Review

Our work reconciles two strands of studies. First, there is a predominantly empirical literature on retail structured products. Second, our analysis is also related to the literature dealing with the capital structure and risk management of firms and especially financial institutions.

The focus of the empirical literature on retail structured products is on the pricing from the investors' perspective. In one of the most comprehensive empirical studies of the German market, Stoimenov and Wilkens (2005) document that retail structured products are traded at a markup compared to their stand-alone duplication values. They attribute this observation to information asymmetries and retail investors' limited market access. Their results are confirmed by many further studies, e.g., Wilkens et al. (2003), Baule, Entrop, and Wilkens (2008), Entrop, Scholz, and Wilkens (2009), and Baule (2011). In addition, Baule et al. (2008) show that the default risk of the issuer is not appropriately reflected in the pricing of retail structured products. Henderson and Pearson (2011) provide similar evidence for equity linked products in the U.S., which are also mainly traded by retail investors.

Carlin's (2009) model supplements this empirical evidence on the pricing of retail structured products. His key result is that producers of financial products can increase the profits they make from selling these products to uninformed retail investors by making the products more complex. Breuer and Perst (2007) make another interesting theoretical contribution. These authors explore why utility-maximizing retail investors want to add retail structured products to their portfolios in the first place. According to their results, the purchase of retail structured products is particularly beneficial for investors with low levels of competence in investing.

Our work also follows the tradition of structural models in corporate finance. Considering typical frictions such as the tax benefits of debt and bankruptcy costs, these models are capable of deriving an optimal capital structure. One of the first models to implement the trade-off between tax benefits and bankruptcy costs is that of Brennan and Schwartz (1978), which builds on the option theoretic approach of Merton (1974). This approach has been further developed in continuous time by Fischer, Heinkel, and Zechner (1989), Leland (1994), and Goldstein, Ju, and Leland (2001). Decamps, Rochet, and Roger (2004) apply the framework to financial institutions and derive implications for the risk-taking incentives and stability of banks.

Following Modigliani and Miller (1958), there is no optimal capital structure in a frictionless world. Similarly, firms cannot add value with risk management. Hence, the need for risk management arises when firms try to avoid the costs related to frictions; for example, the costs of financial distress, which is also the motive for hedging in our model. Froot and Stein (1998) provide an extensive study of the risk management incentives of financial institutions.

So far, the literature has not considered the linkages between these two strands. Since issuers, however, have access to highly sophisticated financing claims such as retail structured products, it is essential to analyze the impact of issuing such products on the issuers risk-taking incentives and stability.

3 Model

3.1 Investment and Financing Choices

We consider an initially unlevered financial institution (issuer) in a one-period setting with initial time $t = 0$ and maturity time $t = T$. The issuer holds an asset portfolio with value $\tilde{A}_t$ at time t. The asset structure remains static until maturity. We consider different compositions of the issuer's asset portfolio.

The financial institution may choose to issue zero coupon bonds and retail structured products (RSPs). The raised capital is immediately paid out as a cash dividend to equity holders. The demand is sufficiently large such that the issuer can raise any desired amount of debt. We focus our analysis on the two most prominent claims, principal-protected notes (PPNs) and discount notes (DCNs). The issuer can issue only one type of product at a time. We do not require a specific seniority structure; we model debt as one claim. Thus, the split among the debtors in the case of default is arbitrary and does not impact the results.

The issuer promises holders of the bond B a fixed payment of $\overline{B}$ at maturity T. The RSP payoff is linked to the performance of an underlying security $\tilde{R}$, for example, a stock market index such as the Euro STOXX 50 or the Dow Jones Industrial Average. The promised payoff of the principal-protected note CP_T at maturity T is given by

$$CP_T = \left(1 + \pi \cdot \max\left\{\frac{\tilde{R}_T - X_P}{X_p}, 0\right\}\right) \cdot \overline{P}, \tag{1}$$

where $\overline{P}$ denotes the minimum payment to investors (see *Figure 13* in Appendix A.1). Investors participate at the rate of π in the performance of the underlying asset above the threshold X_p, which usually matches the initial value of the underlying asset $X_p = R_0$. Hence, the investor is protected against decreases in the underlying value as long as the issuer remains solvent. The promised payoff is equivalent to that of a portfolio comprising a risk-free zero bond with face value $\bar{P}$ and $\pi \cdot \bar{P}$ times a call option with strike price X_P.

The promised payoff of the discount note CD_T at maturity T is given by

$$CD_T = \min\left\{1, \gamma \cdot \tilde{R}_T\right\} \cdot \bar{D}, \tag{2}$$

where $\bar{D}$ denotes the maximum payment to investors (see *Figure 13* in Appendix A.1). We define $\gamma \equiv \frac{1}{X_D}$. If the price of the underlying $\tilde{R}_T$ falls below the threshold X_D, then the investors are paid the value of the underlying asset. This promised payoff can be duplicated with a portfolio consisting of a risk-free zero bond with face value $\bar{D}$ and $\gamma \cdot \bar{D}$ times a short put with strike price X_D.

All market participants have perfect information. Investors observe market prices as well as the structure of the issuer's asset portfolio. They are able to anticipate the issuer's decision and appropriately incorporate the information in the pricing of the claims.

The issuer is operated by managers on behalf of the equity holders. The managers choose the principle amount $\bar{B}$ of the discount bond and the product parameters $\bar{P}$ and $\bar{D}$ to maximize the value of the equity holders' position at time $t = 0$. According to the well known result of Modigliani and Miller (1958), the manager's choice is arbitrary in a world of complete and efficient markets. Hence, we allow for the classical trade-off between tax benefits of debt and bankruptcy cost.

At maturity T, the issuer repays its debt and pays taxes at rate $\tau > 0$. The tax deductibility of interest payments allows the issuer to derive value from debt financing.[2] Similarly, the issuer can derive tax benefits from retail structured products, for which the tax deductible cost of financing is equal to the difference between the repayment and the issuance price. Since the repayment is linked to the underlying asset $\tilde{R}$, the size of the tax shield also depends on the realization of the underlying asset and can possibly turn negative in some states of the world.

The issuer defaults if the value of its debt exceeds the value of its assets. In this case, the debt holders receive a share $1 - \alpha$ of the issuer's assets, where $\alpha \in (0,1]$ denotes the proportional cost of bankruptcy. A potentially positive tax shield is lost.

Alternatively to the tax benefits, we could assume that the issuer has a franchise value, i.e., the capability to generate additional revenues from business related to issuing retail structured products. Such revenues include fees for sales, trading, and depository of the

2 We do not consider the personal income tax of the equity holders and debt holders. Their effect is negligible if all investors pay the same tax rate on dividends, interest income, and gains in the value of traded securities, which has been the case in Germany since 2009.

securities. We analyze such a setup in Section 7.4. Hence, our model framework can accommodate a wide spectrum of market frictions.

3.2 Valuation of Claims

We build on the approach of Merton (1974), who interprets the equity holders' payoff at maturity T as a call option on the issuer's assets with the issuer's liabilities corresponding to the strike price. Hence, the established valuation framework for contingent claims can be applied. Our model differs in one dimension: the issuer's liability at maturity T, i.e., the strike price of the option, is itself contingent on the price of a risky asset.

We follow the set of assumptions provided by Black and Scholes (1973) and Merton (1973).[3] The price of the underlying asset $\tilde{R}$ follows a diffusion process of the form

$$\mathrm{d}R = \mu_R R_t \,\mathrm{d}t + \sigma_R R_t \,\mathrm{d}z_R, \tag{3}$$

where μ_R denotes the underlying asset's expected rate of return, σ_R denotes the standard deviation of returns, and z_R is a Wiener process. The underlying asset $\tilde{R}$ is not paying a dividend. The term structure of interest rates is constant and flat. The value of the risk-free asset F_t at any point in time t is determined by the risk-free interest rate r with

$$F_t = F_0 \cdot e^{rt}. \tag{4}$$

We consider two settings for the asset value $\tilde{A}$. In the most general case (see Section 5), the asset value also follows a diffusion process of the form

$$\mathrm{d}A = \mu_A A_t \,\mathrm{d}t + \sigma_A A_t \,\mathrm{d}z_A, \tag{5}$$

where μ_A denotes the asset's expected rate of return, σ_A denotes the standard deviation of returns, and z_A is a Wiener process, which is correlated to the Wiener process z_R determining the value of the underlying, i.e., $\mathrm{d}z_R\,\mathrm{d}z_A = \rho\,\mathrm{d}t$ with $\rho \in (-1,1)$. Using risk-neutral valuation, the value of the issuer V_0 at time $t = 0$ equals

$$V_0 = D_0 + e^{-rT} \int_0^\infty\!\!\int_0^\infty \left(A_T - D_T + \tau\left(D_T - D_0\right)\right) \cdot \mathbb{1}_{\text{solvency}} \cdot f_{RA}\left(R_T, A_T\right) \mathrm{d}R_T \,\mathrm{d}A_T, \tag{6}$$

3 With the exception of taxes and bankruptcy cost, the market is free of frictions. There are no transaction costs or bid-ask-spreads. Trading in the underlying asset is continuous and all securities are perfectly divisible. All market prices are observable and short selling is not restricted. Investors are assumed to be non-satiable and agree on σ, but not necessarily μ.

where D_t denotes the value of total debt including retail structured products at time t, and $f_{RA}(R_T, A_T)$ is the joint risk-neutral probability density function of the underlying asset R_T and the issuer's asset value $\tilde{A}_T$ at time T. The indicator function $\mathbb{1}_{\text{solvency}}$ for the survival of the issuer takes the value of one for $A_T - D_T \geq 0$ and zero otherwise.

When the issuer is able to fully repay the debt, it generates a tax benefit with present value $\tau(D_T - D_0)e^{-rT}$. The tax benefit is lost if the issuer defaults. The bankruptcy cost are included in the pricing of the debt claim D_0. The value V_0 is given by the value of the assets A_0 of the unlevered issuer plus the present value of the tax-shield minus the present value of the bankruptcy cost.

In a simplified setting (see Section 6), we consider only one single source of uncertainty. In this case, the asset portfolio $\tilde{A}$ of the issuer is linked to the development of the underlying asset $\tilde{R}$. The expression of the issuer value V_0 simplifies to

$$V_0 = D_0 + e^{-rT}\int_0^\infty \left(A_T - D_T + \tau\left(D_T - D_0\right)\right)\mathbb{1}_{\text{solvency}} \cdot f_R\left(R_T\right)\mathrm{d}R_T. \tag{7}$$

We can derive a closed-form solution for the equity holders' claim V_0 (issuer value). The functions are piecewise defined depending on the managers' choice of $\bar{B}$, $\bar{P}$, and $\bar{D}$. To improve readability, we present the exhaustive derivation of the formulae in Appendix A.2.

We introduce a measure for the stability of the issuer. For this purpose, we use the risk-neutral probability of default pd, which we calculate as

$$pd = \int_0^\infty\int_0^\infty \left(1-\mathbb{1}_{\text{solvency}}\right)\cdot f_{RA}\left(R_T, A_T\right)\mathrm{d}R_T\,\mathrm{d}A_T. \tag{8}$$

Since the quotes of credit default swaps written on the issuer monotonically increase with the risk-neutral default probability, pd is a reasonable market-oriented measure for stability.

4 Constant Leverage

Before we take a look at the optimal financing and risk choices, we inspect the issuer value and the probability of default depending on the leverage ratio $\lambda = \frac{D_0}{V_0}$. By doing so we can draw important conclusions on the value generated by RSPs and on the stability of the issuer. We focus on two polar cases. First, we consider a high-risk issuer whose assets are the same as the underlying asset of the RSP, i.e.,

$$\tilde{A}_t = \tilde{R}_t. \tag{9}$$

Second, we analyze a low-risk issuer investing only in risk-free government bonds, i.e.,

$$\tilde{A}_t = F_t. \tag{10}$$

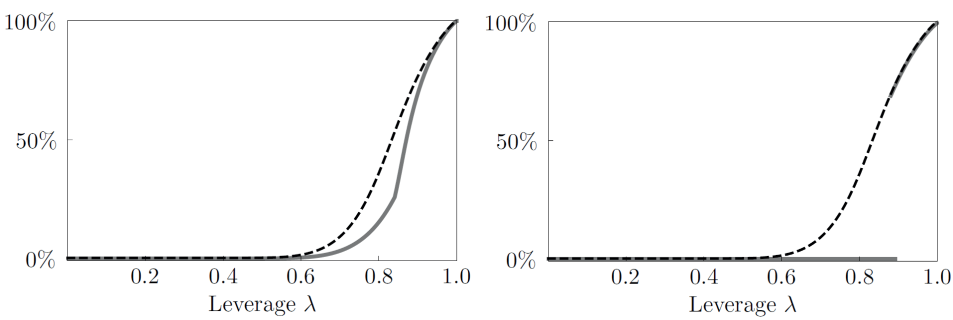
In addition, we restrict the issuer to issuing one single debt claim. This approach has the advantage that the valuation formulae simplify and general results can be derived analytically.

4.1 High-Risk Issuer

We first consider the case of $\tilde{A}_t = \tilde{R}_t$. On the one hand, this case represents an issuer taking the maximum amount of risk. On the other hand, this issuer also has the greatest capability to produce RSPs, which depend on the same risky asset that is part of the issuer's balance sheet.

Before analyzing the issuer value, we examine the risk-neutral default probability of the issuer, which is depicted in *Figure 1*. The graph on the left shows the default probability of an issuer financed with PPNs (solid line) and the graph on the right shows the default probability of an issuer financed with DCNs (solid line). Both plots also show the default probability under straight debt financing as a reference case (dashed line).

Figure 1: Probability of Default Depending on Leverage (High-Risk)



The graph on the left shows the probability of default pd_{PPN} for an issuer with PPN financing (solid line). The graph on the right shows the default probability pd_{DCN} for an issuer with DCN financing (solid line). Both graphs also show the default probability pd_B with straight debt financing (dashed line). We compute the values using the model parameters $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, and $\alpha = 0.25$ and product parameters $X_p = 100$, $\pi = 0.5$, and $X_D = 125$.

In line with our expectations, the curves monotonically increase with the leverage ratio λ. For PPN financing, we have to distinguish two cases. For low issuance volumes $\overline{P} < X_P$, the issuer defaults only when the value of the underlying asset drops below the issued principal amount, i.e., $R_T < \overline{P}$. But when the issued amount $\overline{P}$ exceeds X_P, the issuer is also not able to repay the promised participation in the underlying asset even though the value of the underlying asset appreciates. *Figure 1* shows that the graph has a kink at the transition point between these two cases at $\overline{P} = X_P$.

For an issuer with DCN financing, we also observe two cases. The issuer can repay its liabilities in all states of the world as long as the issued amount $\overline{D}$ is less than the maximum repayment X_D, i.e., we have $pd_{DCN} = 0$. However, the default probability jumps up when $\overline{D}$ exceeds X_D, since the issuer is defaulting for all values of the underlying asset, $R_T < \overline{D}$. In this case, the default probability corresponds to that of an issuer with straight debt financing with an issued amount $\overline{B} = \overline{D}$.

The main finding from *Figure 1* is that the probability of default with RSP financing is either equal to or strictly lower than the default probability of an issuer with straight debt financing. This observation can be generalized due to the closed form solutions for all claim values. We provide proofs in Appendix A.3[4].

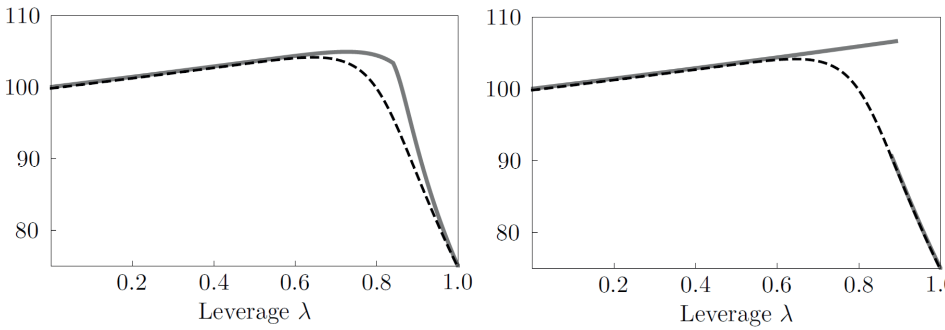
We summarize this important result as:

Proposition 1 (Risk reduction of high-risk issuer): *For any attainable leverage ratio $\hat{\lambda} < 1$, the risk-neutral default probability of a high-risk issuer financed with RSPs never exceeds the probability of default of a high-risk issuer financed with straight debt, i.e., $pd_{RSP}(\hat{\lambda}) \leq pd_B(\hat{\lambda})$.*

The next logical step in our analysis is to consider the issuer value, which is depicted in *Figure 2*. The graph on the left shows the value of an issuer financed with PPNs (solid line) and the graph on the right shows the value of an issuer financed with DCNs (solid line). Both plots also show the issuer value for straight debt financing as a reference (dashed line).

4 The proof for DCN requires the technical condition $N\left(d_2\left(y\right)\right) - N\left(d_1\left(y\right)\right) \leq \overline{\varepsilon}$ for all y. The proof for PPN requires $\overline{P} < \frac{X_P}{\pi}$ for $\pi > 1$.

Figure 2: Issuer Value Depending on Leverage (High-Risk)



The graph on the left shows the issuer value $V_{0,PPN}$ with PPN financing (solid line). The graph on the right shows the issuer value $V_{0,DCN}$ with DCN financing (solid line). Both graphs also show the issuer value $V_{0,B}$ with straight debt financing (dashed line). We compute the values using the model parameters $A_0 = 100$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, and $\alpha = 0.25$ and product parameters $X_P = 100$, $\pi = 0.5$, and $X_D = 125$.

The issuer value increases with the leverage ratio λ and then decreases to $(1-\alpha)A_0$ when λ approaches one. This behavior is consistent with the results of Leland (1994). Analogous to the corresponding graph of the default probability, the issuer value under PPN financing has a kink at $\bar{P} = X_D$. Due to the zero default probability, the issuer value under DCN financing increases linearly until $\bar{D} = X_D$ and then drops down to the issuer value under straight debt financing.

We observe that the issuer value under RSP financing is always equal to or higher than the value under straight debt financing. Again, we can generalize this important result. (See Appendix A.3 for proof.)

Proposition 2 (Value creation of high-risk issuer): *For any attainable leverage ratio $\hat{\lambda} < 1$, the value of a high-risk issuer financed with RSPs is always greater than or equal to the value of a high-risk issuer financed with straight debt, i.e., $V_{0,RSP}(\hat{\lambda}) \geq V_{0,B}(\hat{\lambda})$.*

In summary, the high-risk issuer always benefits from the issuance of RSPs. Propositions 1 and 2 show that the issuer can increase its value and at the same time reduce the probability of default for fixed leverage ratios as compared to the case of straight debt financing.

Surprisingly, this result holds for both types of products, i.e., concave payoffs as well as convex payoffs. The benefit of PPNs compared to straight debt financing is that given the same probability of default, PPNs can create higher tax benefits. This increase in tax benefits is achieved by selling a fraction of the assets only in good states $\tilde{R}_T > X_P$ at maturity time T. In contrast, the benefit of DCNs financing originates from a lower repayment to debt holders in bad states $\tilde{R}_T < X_D$ at maturity, which allows the issuer to reduce its expected bankruptcy cost compared to straight debt financing.

This result is certainly only valid for a fixed leverage ratio. It is apparent from *Figure 2* that the optimal leverage for RSP financing is higher than that for straight debt financing. We analyze this optimal choice in more detail in Section 6. Nevertheless, we can still derive an important implication here for the regulator. Due to the one-to-one correspondence between the leverage ratio and probability of default, the regulator can easily impose restrictions on the leverage to fit the maximum amount of risk that the issuer should take from the social planner's perspective.

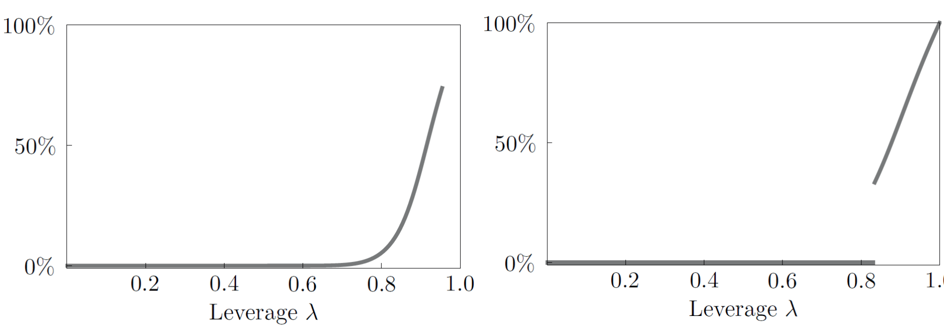
4.2 Low-Risk Issuer

The issuer with $\tilde{A}_t = \tilde{R}_t$ considered so far is well capable to issue RSPs, due to the high exposure to the risky underlying on the balance sheet. In this section, we evaluate the opposite case of an issuer with no exposure to the risky underlying asset. The assets of the issuer characterized by $\tilde{A}_t = F_t$ are completely free of risk. This asset structure implies that the issuer could borrow a face value up to $F_T = A_0 \cdot e^{rT}$ at the risk-free rate.

Again, we first examine the risk-neutral probability of default. The case of straight debt financing is apparently simple. As long as the face value of the bond $\overline{B}$ is lower than the asset payoff F_T, the default probability is zero. If more debt is issued, then both the leverage ratio and the default probability increase to one.

Figure 3 illustrates the default probability of RSP issuers. The graph on the left shows the default probability of an issuer financed with PPNs and the graph on the right shows the default probability of an issuer financed with DCNs.

Figure 3: Probability of Default Depending on Leverage (Low-Risk)



The graph on the left shows the probability of default pd_{PPN} for an issuer with PPN financing. The graph on the right shows the default probability pd_{DCN} for an issuer with DCN financing. We compute the values using the model parameters $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, and $\alpha = 0.25$ and product parameters $X_P = 100$, $\pi = 0.5$, and $X_D = 125$.

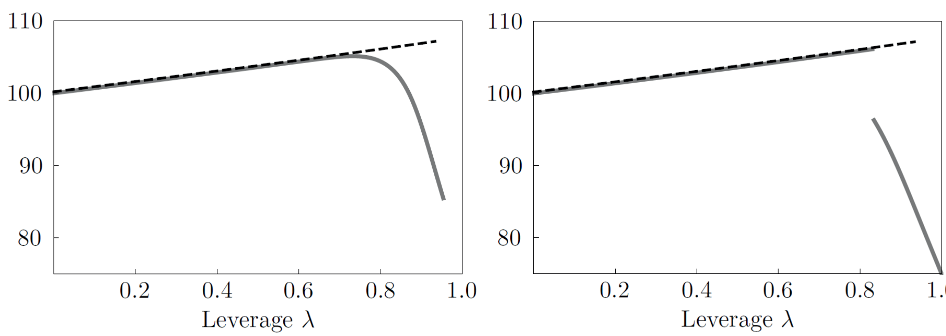
The default probability of the PPN issuer increases monotonically as long as $\bar{P} \leq F_T$. The issuer defaults for high values of the underlying asset. When more debt is issued, i.e., for $\bar{P} > F_T$, the default probability rises to one. The DCN issuer does not default as long as $\bar{D} \leq F_T$. For higher debt volumes of $\bar{D} > F_T$, the default probability jumps up and tends to one, as the issuer is now defaulting for high realizations of the underlying asset $\tilde{R}_T > \frac{F_T}{\gamma \bar{D}}$.

Since the issuer of straight debt never defaults for $\lambda < 1$, the issuer of RSP is always worse off. The low-risk issuer introduces a dependency to the risky asset by issuing RSPs. This dependency increases the probability of default for some leverage ratios, but can never decrease it. This result again can be generalized. (See Appendix A.3 for proof.)

Proposition 3 (Risk increase of low-risk issuer): *For any attainable leverage ratio* $\hat{\lambda} < 1$, *the risk-neutral probability of default of a low-risk issuer financed with RSPs is always greater than or equal to the default probability of a low-risk issuer financed with straight debt, i.e.,* $pd_{RSP}(\hat{\lambda}) \geq pd_B(\hat{\lambda})$.

This result at first seems problematic from the regulator's point of view, since he is naturally concerned about increasing default probabilities. But to evaluate if the issuer actually prefers to issue RSPs over straight debt, we again need to inspect the issuer value. The corresponding issuer values are depicted in *Figure 4*. The graph on the left shows the value of an issuer financed with PPNs (blue solid line) and the graph on the right shows the value of an issuer financed with DCNs (red solid line). Both plots also show the issuer value under straight debt financing as a reference case (dashed line).

Figure 4: Issuer Value Depending on Leverage (Low-Risk)



The graph on the left shows the issuer value $V_{0,PPN}$ with PPN financing (solid line). The graph on the right shows the issuer value $V_{0,DCN}$ with DCN financing (solid line). Both graphs also show the issuer value $V_{0,B}$ with straight debt financing (dashed line). We compute the values using the model parameters $A_0 = 100$, $\sigma_R = 0.2$, $\tau = 0.15$, $T = 1$, $r = 0.5$, and $\alpha = 0.25$ and product parameters $X_P = 100$, $\pi = 0.5$, and $X_D = 125$.

The issuer value under straight debt financing increases linearly with leverage λ, since tax benefits can be generated at no additional cost. In contrast, the RSP issuer incurs an additional bankruptcy cost when the probability of default rises. Consequently, the issuer value with RSP financing lies below the value under straight debt financing whenever there is a positive default probability. The issuer value with PPN financing decreases for high leverage ratios up to $\bar{P} = X_P$. The issuer value with DCN financing agrees with the value under straight debt financing up to $\bar{D} = F_T$. It drops down and decreases towards $(1 - \alpha)A_0$ when the face value $\bar{D}$ is further increased. We summarize this important result in the following proposition. (See Appendix A.3 for proof.)

Proposition 4 (Value destruction of low-risk issuer): *For any attainable leverage ratio* λ $\hat{\lambda}<1$, *the value o f a low-risk issuer financed with RSPs never exceeds the value of a low-risk issuer financed with straight debt, i.e.,* $V_{0,RSP}\left(\hat{\lambda}\right) \leq V_{0,B}\left(\hat{\lambda}\right)$.

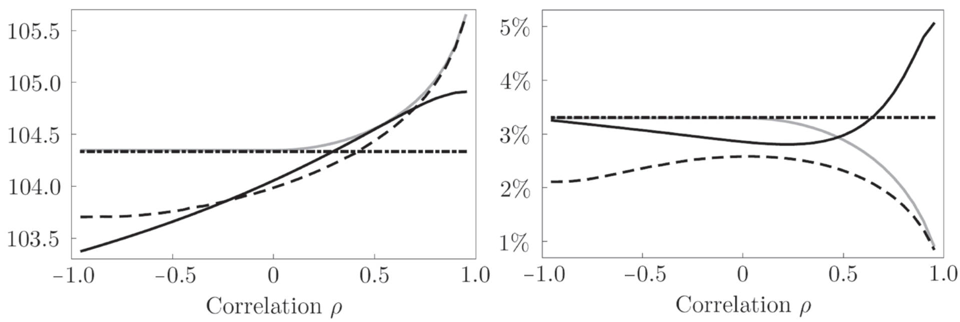
We conclude from Propositions 3 and 4 that low-risk issuers never benefit from the issuance of RSPs. The highest tax benefits are generated by issuing risk-free debt. In contrast, the issuance of RSPs may increase the default probability. In these cases, the bankruptcy costs eat up the tax benefits. Again, this result holds for both types of contracts, i.e., for concave as well as for convex payoff structures. The regulator does not have to consider the danger of RSP financing for low-risk issuers, since they do not voluntarily issue them.

We conclude that the benefits of issuing RSPs depend critically on the risk of the issuer's asset portfolio. The high-risk issuer can use RSPs to reduce the probability of default, i.e., as a form of insurance. The low-risk issuer has no need for insurance. Thus, RSPs have the opposite effect in this case. They increase the riskiness of the issuer, since they introduce a dependency on the risky underlying asset.

5 Optimal Financing Choice

In the next step, we evaluate the impact of RSPs on the optimal financing choice of the bank. To accommodate a more general and realistic set of scenarios, we assume that the bank's assets and the underlying asset of the RSPs are not the same. Hence, the asset portfolio is exogenous and has a constant volatility σ_A. However, the returns of the assets and the underlying asset are correlated with coefficient $\rho \in \left(-1,1\right)$. A perfect correlation of $\rho = 1$ corresponds to the high-risk issuer described in Section 4.1. We resort to numerical solutions for the claim values in this section.

Figure 5: Issuer Value and Default Probability Depending on Correlation



The plot on the left shows the optimal issuer value V_0 with PPN financing (solid black line), with DCN financing (dashed line) and with straight debt financing (dot-dashed line) depending on the correlation ρ. The plot on the left also shows the optimal value when the issuer can finance itself with any mix of straight debt and DCNs (solid gray line). The plot on the right shows the corresponding risk-neutral default probabilities. We compute the values using the model parameters $A_0 = 100$, $\sigma_A = 0.2$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, and $\alpha = 0.25$ and product parameters $X_P = 100$, $\pi = 0.5$, and $X_D = 125$.

We first inspect the issuer value V_0 which depends on the correlation ρ between the returns of the assets and the underlying asset (see left-hand plot of *Figure 5*). The value of an issuer financed with only straight debt (the dot-dashed line) is obviously independent of the correlation, since there is no link to the risky underlying asset. The issuer values under PPN financing (solid black line) and under DCN financing (dashed line) both increase with the correlation. For negative and low positive correlations, financing with RSPs reduces the issuer value compared to straight debt financing. Since the payoff of both products increases in the value of the underlying asset, financing with RSPs is only beneficial when the correlation is high, i.e., when the values of the issuer's assets and the underlying security behave similarly.

This finding is confirmed when we examine a mix of different debt contracts. The issuer value for a financing mix consisting of straight debt and DCNs is also depicted in the left-hand plot of *Figure 5* (solid gray line). For negative correlations, the issuer uses only straight debt. However, the issuer always adds a strictly positive fraction of RSPs to the financing mix when the correlation turns positive. The weight of RSPs in the financing mix increases monotonically with the correlation up to a share of 100%. The results for a financing mix which includes PPNs (not shown) are qualitatively the same.

Next, we evaluate the impact of RSP financing on the default risk of the issuer. We plot the risk-neutral probability of default *pd* depending on the correlation ρ on the right-hand side of *Figure 5*. Again, the default risk of an issuer financed with only straight debt is independent of the correlation. PPN financing (solid black line) turns out to reduce the default probability of the issuer for low positive and for negative correlations. However, it is not optimal to finance with PPNs for those correlations. But PPN financing increases default risk for high correlations, when PPN have an advantage over straight debt in terms of value maximization.

In contrast, for all possible correlations DCN financing reduces the default risk of the issuer compared to straight debt financing. The effect is also large in magnitude. For example, the default risk is reduced from 3.3% to 0.9% for a correlation of $\rho = 1$. The hedging benefit is still present when we examine an optimal mix of DCNs and straight debt (solid gray line). DCNs are not added to the financing mix for negative correlations. The default risk of the issuer declines with an increasing share of DCNs in the financing mix and thus with an increasing correlation. The issuer could further decrease the default probability by issuing only DCNs, but doing so is not optimal in terms of value maximization.

Clearly, adding RSP to the financing mix is always beneficial for positive correlations between the assets and the underlying. When issuing DCNs, the issuer can thereby reduce its default probability. In contrast, PPN financing causes the default risk of the issuer to increase when the correlation is high. A comparative static analysis of these results is contained in Appendix A.4.

6 Optimal Risk-Taking

We have shown in Section 4 that high-risk issuers prefer RSP over straight debt. Low-risk issuers prefer the opposite. Furthermore, we have shown in Section 5 that issuers optimally add RSP to their financing mix whenever the correlation between the assets and the underlying is positive. In this section, we tackle the question of how the issuer's choice of asset risk is influenced when RSPs are available as an instrument for financing and risk management.

For this purpose, we consider an asset portfolio that is a linear combination of the underlying $\tilde{R}$ with weight $\delta \in [0,1]$ and the risk-free asset F with weight $1 - \delta$. Thus, the asset value $\tilde{A}_t$ of the unlevered issuer at time t is given by

$$\tilde{A}_t = \delta \cdot \tilde{R}_t + (1-\delta) \cdot F_t. \quad (11)$$

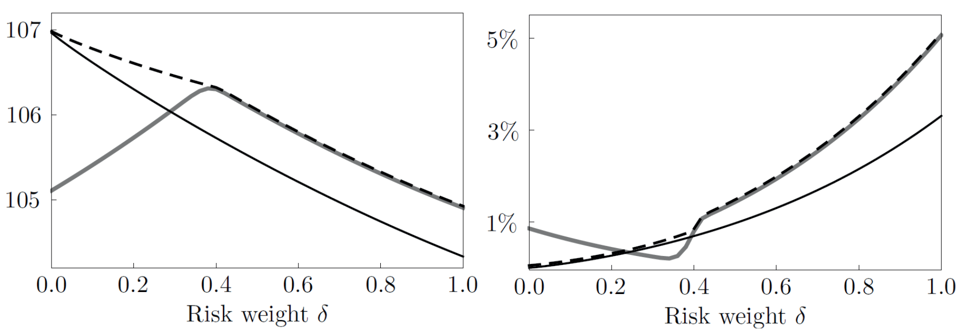
The financial institution trades in securities, lends money to consumers and enterprises, purchases government bonds, and holds central bank deposits. We assume that all such investments are separable into a component impacted by the source of uncertainty $\tilde{R}$ and a residual component F, which is free of risk. The high-risk and low-risk issuers discussed in Section 4 are represented by $\delta = 1$ and $\delta = 0$, respectively. The asset structure described here corresponds to the case $\rho = 1$ discussed in the previous Section 5, i.e., the case in which RSPs add most value. However, the volatility of the assets is no longer constant. The parameter δ scales the volatility of the assets such that $\sigma_A = \delta \cdot \frac{R_t}{A_t} \cdot \sigma_R$.

In the following, we analyze the issuer's optimal financing choice for a given asset risk weight δ as well as the optimal risk weight choice. In Section 6.3, we consider the risk-shifting incentives of equity holders. We use numerical optimization techniques, since solutions for the optimal decisions cannot be obtained in closed form. We control the optimization results for many different scenarios. The comparative static analysis can be found in Appendix A.4.

6.1 Principal-Protected Notes

The issuer can finance with straight debt, PPNs, or a mix of both. We determine the optimal leverage ratio λ^* for each risk weight δ. *Figure 6* shows the resulting optimal issuer values on the left-hand side and the corresponding probability of default given the optimal leverage on the right. The graphs show the values for the issuer financed with straight debt (thin black line), for an issuer financed with PPNs only (thick gray line), and for an issuer financed with a mix of bonds and PPNs (dashed line).

Figure 6: Optimal Issuer Value and Probability of Default with PPN Financing



The graph on the left shows the optimal issuer value. The graph on the right depicts the probability of default given the optimal leverage. The plots show the values for an issuer financed with straight debt (thin black line), for an issuer financed with PPN only (thick gray line) and for an issuer financed with a mix of bonds and PPN (dashed line). We compute the values using the parameters A_0 = 100, σ_R = 0.2, r = 0.15, T = 1, τ = 0.5, α = 0.25, X_P = 100, and π = 0.5.

We look first at an issuer with one single debt claim outstanding. The optimal value with straight debt financing strictly decreases with the asset risk weight δ. The maximum is at $\delta = 0$. The probability of default pd_B strictly increases with δ from zero for $\delta = 0$ up to 3.3% for $\delta = 1$. These findings reproduce the well known results of Merton (1974) and Leland (1994). We use this case as a reference to evaluate the impact of RSP financing.

We can reconcile the results shown in *Figure 6* with the findings from Section 4. The optimal value of a low-risk issuer with $\delta = 0$ financed with RSPs is below the value of the issuer financed with straight debt. The opposite is true for a high-risk issuer with $\delta = 1$. Hence, there must be an asset risk weight for which the issuer is indifferent between financing with bonds and RSPs. For the chosen parameter values, this risk weight is approximately at $\delta = 0.29$. For lower risk weights, the issuer prefers to finance with straight debt. For higher risk weights, the issuer prefers to finance with PPNs.

The optimal issuer value under PPN financing is a hump-shaped curve with its maximum at $\delta = 0.39$. For all tested scenarios, the maximum issuer value with PPN financing never exceeds the maximum value when the issuer uses straight debt. Given the optimal choice, the corresponding default probability decreases for low risk weights and increases sharply around the maximum issuer value at $\delta = 0.39$, thereby surpassing the default probability under straight debt financing. It continues to increase up to the maximum of 5.1% for $\delta = 1$. At first, this finding seems to contradict proposition 1, which states that the default probability with PPN financing should be reduced compared to straight debt financing. However, the issuer has an incentive to optimally increase the leverage ratio λ. In the case of PPN financing, this increase in leverage eats up the beneficial effect of RSP on the default probability.

We next consider an issuer who can choose any arbitrary mix of zero bonds and PPNs to finance itself. Since this financing mix adds an extra degree of freedom to the optimization, the issuer can never be worse off compared to the case of a single debt claim.

The most important finding is that the issuer always chooses to finance itself with a positive amount of PPNs for all positive risk weights $\delta > 0$. The low-risk issuer with $\delta = 0$ finances itself with straight debt only as shown in Propositions 3 and 4. The issuer combines bonds and principal-protected notes for $0 < \delta < 0.39$. For higher risk weights, the issuer relies only on PPNs for financing. The resulting curve for the issuer value is a monotonically decreasing function in the risk weight δ. The maximum is at $\delta = 0$, i.e., the case of straight debt financing. We also observe that given optimal leverage, the probability of default is always equal to or higher than the default probability of the bond financed issuer.

Finally, we consider the choice of the optimal risk weight δ^*. An issuer always has the incentive to reduce the risk weight as much as is feasible, i.e., an issuer with full flexibility chooses a risk weight of $\delta = 0$. This is good news for the regulator, since the default probability at the optimum is zero. However, should the issuer be constrained from further reducing the risk weight, the regulator might be concerned in two cases. In the first case, when the minimum attainable risk weight is below 0.39, the issuer optimally chooses a mix between straight debt and PPNs. However, a financing with only PPNs would result in a lower probability of default. In the second case, when the minimum attainable risk weight is above 0.39, the issuer relies only on PPNs for financing. Again, a lower probability of default can be achieved by financing with straight debt only.

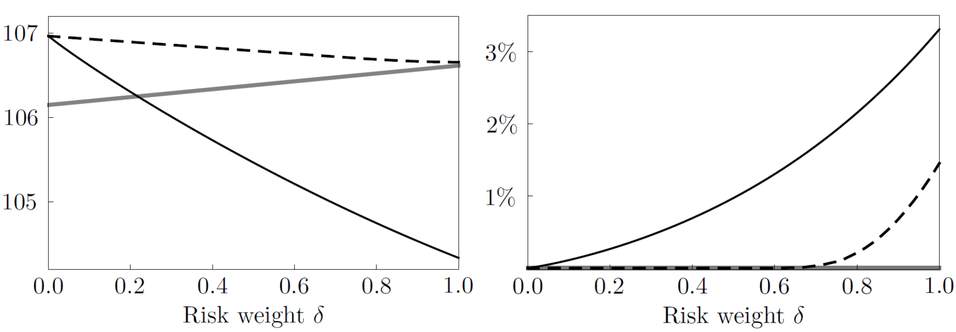
As noted earlier, the regulator can exploit the one-to-one relation between the default probability and the leverage ratio to limit the risk taking incentive of the issuer. Unfortunately, this relation changes fundamentally with the risk weight δ. For example, if we look at the issuer value under PPN financing, the value maximizing leverage ratio at $\delta = 0.39$ is higher than the leverage ratio at $\delta = 1$. However, the resulting probability of default at $\delta = 1$ is more than five times as high. Hence, the maximum leverage ratio prescribed by the regulator should either incorporate the asset risk of the issuer or it should be geared towards the worst-case scenario, i.e., $\delta = 1$.

We conclude that adding PPN to the financing mix of the issuer can increase the issuer value. However, there is also the danger that the default risk of the issuer increases. This increase is especially severe for an issuer who inherits a high exposure to the risky asset and is either not capable of adjusting this exposure in the short run or incurs a high cost when doing so.

6.2 Discount Notes

Next, we analyze an issuer who is financed with straight debt, DCNs, or a mix of both. Again, we determine the optimal leverage ratio λ^* for each risk weight δ. *Figure 7* depicts the resulting optimal issuer values on the left-hand side and the corresponding probability of default at the optimum on the right. The graphs show the values for the issuer financed with straight debt (thin black line), for an issuer financed with DCNs only (thick gray line), and for an issuer financed with a mix of bonds and DCNs (dashed line).

Figure 7: Optimal Issuer Value and Probability of Default with DCN Financing



The graph on the left shows the optimal issuer value. The graph on the right depicts the probability of default given the optimal leverage choice. The plots show the values for an issuer financed with straight debt (thin black line), for an issuer financed with DCN only (thick gray line) and for an issuer financing with a mix of bonds and DCN (dashed line). We compute the values using the parameters $A_0 = 100$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, $\alpha = 0.25$, and $X_D = 125$.

We first analyze an issuer with one single debt claim outstanding. The optimal value with straight debt financing is still our reference scenario. Moreover, *Figure 7* mirrors the results from Section 4. The optimal value of a low-risk issuer with $\delta = 0$ financed with RSPs is below the value of the issuer financed with straight debt. The opposite is true for a high-risk issuer with $\delta = 1$. The issuer is indifferent at a risk weight of approximately $\delta = 0.22$. For lower risk weights, the issuer prefers to finance with straight debt. For higher risk weights, the issuer prefers to finance with DCNs.

The issuer value under DCN financing increases linearly with the risk weight δ. The maximum is at $\delta = 1$. Remarkably, the default probability drops to zero for all risk weights δ. This drop is due to the DCNs' insurance property discussed earlier. The issuer reduces the repayment in bad states of the world and consequently lowers both the default risk and the expected bankruptcy costs.

When the issuer can mix the two debt claims, it chooses to issue a positive amount of DCNs for all positive risk weights $\delta > 0$. The curve slightly decreases, i.e., the optimal risk weight δ^* is again zero. The corresponding default probability is zero for risk weights lower than 0.7 and then increases monotonically up to 1.5% for $\delta = 1$. The default probability at $\delta = 1$ is positive, since the issuer optimally includes a small but positive fraction of straight debt in the financing mix. Most importantly, the probability of default, given the optimal leverage, is always lower than in the case of straight debt financing. Hence, the issuance of DCNs is desirable from the regulatory point of view and should be actively encouraged.

In short, low-risk issuers with $\delta = 0$ optimally issue bonds, and risky issuers, i.e., $\delta > 0$, prefer to add RSPs to the financing mix. Issuers with high asset risk thereby increase the probability of default when issuing PPNs and reduce it by issuing DCNs, compared to the benchmark case of straight debt financing.

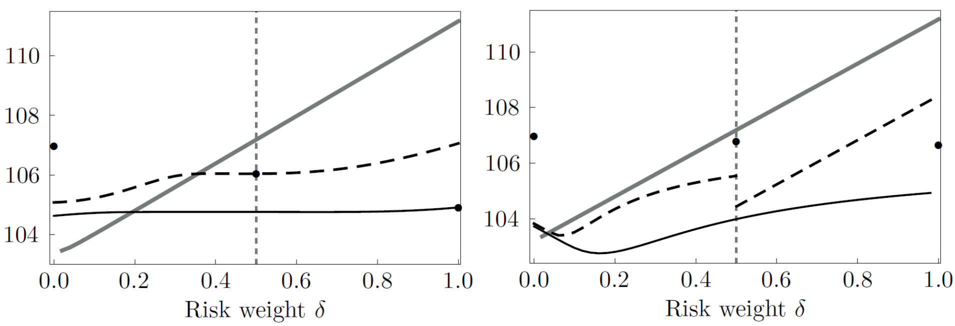
6.3 Risk-Shifting Incentives

We have shown that unconstrained issuers prefer to reduce their asset risk weight to $\delta = 0$. This result implies that only straight debt is used and RSPs are not issued. Only an issuer constrained in the choice of the asset risk weight adds RSP to the financing mix. On the one hand, the issuer might voluntarily keep an exposure to the underlying security, for example, as inventory for trading or due to related businesses. On the other hand, the issuer might not be able to adjust the asset risk weight – at least, not in the short run – due to liquidity constraints or transactions costs.

In addition, the equity holders might not behave optimally in terms of firm value maximization when it is possible to adjust the asset risk weight *after* debt is issued. Our model considers an initially unlevered issuer. The issuer pays out the value of issued debt as a special dividend to equity holders. This setup ensures that if asset risk is contractible, then equity holders maximize the total value of the firm, i.e., the sum of debt and equity value. In the Merton model, the equity value of a levered firm can be thought of as a call option on the firm's assets with the face value of debt corresponding to the strike price. The value of the call option increases with the volatility of the underlying asset. Hence, once debt is issued, equity holders have an incentive to increase the asset risk.

Figure 8 shows the total shareholder wealth when the equity holders engage in risk-shifting behavior. The issuer determines the face value of debt and the debt value, which is paid as a special dividend to equity holders, based on an initial risk weight $\hat{\delta}$. After debt is issued, equity holders are able to adjust the asset risk weight from $\hat{\delta}$ to δ. Debt holders do not anticipate this behavior. The plot shows the resulting optimal shareholder wealth, i.e., the sum of the special dividend given $\hat{\delta}$, and the equity value at the final risk weight δ, for financing with a mix of straight debt and PPNs on the left and for a mix of straight debt and DCNs on the right. We consider three different initial risk weights: $\hat{\delta} = 0$ (thick gray line), $\hat{\delta} = 0.5$ (dashed line) and $\hat{\delta} = 1$ (thin black line). The graphs show the respective values for $\delta = \hat{\delta}$ as black dots, since the functions are not continuous.

Figure 8: Issuer Value and Risk-Shifting Incentives



The plot on the left shows the total shareholder wealth when financing with a mix of straight debt and PPN allowing for a change of the risk weight δ after issuance. Debt is issued assuming an initial risk weight of $\hat{\delta} = 0$ (thick gray line), $\hat{\delta} = 0.5$ (dashed line) or $\hat{\delta} = 1$ (thin black line). The plot on the right shows the total shareholder wealth when financing with a mix of straight debt and DCN for the same initial risk weight scenarios. The graphs show the respective values for $\delta = \hat{\delta}$ as black dots since the functions are not continuous. We compute the values using the parameters $A_0 = 100$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, $\alpha = 0.25$, $X_P = 100$, $\pi = 0.5$, and $X_D = 125$.

The low-risk issuer with $\hat{\delta} = 0$ is financed only with straight debt. The issuer has an initial default probability of zero. Increasing the asset risk weight causes the default probability to increase, which leads to a drop in value. Shareholder wealth increases linearly with the final risk weight δ and surpasses the initial value for risk weights of $\delta > 0.47$. The maximum at $\delta = 1$ results in a shareholder wealth of 111.16 compared to an initial value of 106.94. Similarly, an issuer with initial risk weight $\hat{\delta} = 0.5$ is willing to increase the asset risk weight up to $\delta = 1$. However, the magnitude of the effect is not as large as it is for the low-risk issuer. In contrast, an issuer with initial risk weight $\hat{\delta} = 1$ is not willing to reduce the risk weight. Hence, risk-shifting is beneficial for the issuer, who in all three cases chooses a final risk weight of $\delta = 1$.

Because the risk-shifting phenomenon is well known, we assert that debt holders anticipate the behavior of the issuer. Risk-shifting is to the disadvantage of debt holders, since the subsequent increase in bankruptcy costs reduces the value of debt. Hence, debt holders value their claims as if the issuer chooses an initial risk weight of $\hat{\delta} = 1$. A lower choice of asset risk weight by the issuer is not credible. Consequently, equity holders determine their optimal response for an initial risk weight $\hat{\delta} = 1$ and adjust the debt mix accordingly. This debt mix includes RSPs, since RSPs are added to the financing mix for all positive risk weights $\delta > 0$.

7 Optimal Product Design

Although our focus has been on RSPs with exogenous product properties, we note that these parameters are mainly determined by the preferences of the retail investors. However, the properties of the offered product are, at least to some extent, at the discretion of the issuer. Hence, we analyze what set of parameters the issuer optimally chooses and how this choice affects issuer value and default probability.

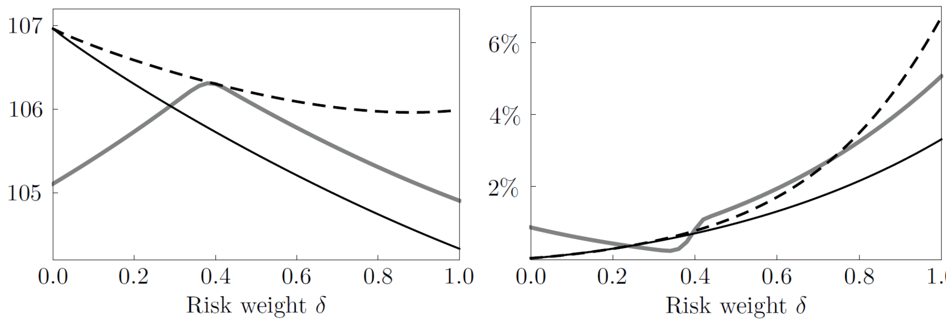
7.1 Principal-Protected Notes

We first focus on an issuer financed only with PPNs. As before, the guaranteed amount is fixed to the level $X_P = R_0$. The issuer optimizes over two parameters, the face value $\overline{P} \geq 0$ and the participation rate $\pi \geq 0$.

The optimal participation rate π^* nearly linearly increases with the risk weight δ, whereby we always observe $\pi^* \cdot \overline{P}^* > \delta$. The issuer defaults for high realizations of the underlying $\tilde{R}_T$. The optimal parameter values range from $\pi^* = 0$ for a low-risk issuer with $\delta = 0$ up to $\pi^* = 1.33$ for a high-risk issuer with $\delta = 1$.

Figure 9 shows the optimal issuer value on the left and the corresponding probability of default on the right. Both plots present the values for an issuer financed with the standard PPN contract with $\pi = 0.5$ (thick gray line) and the values for the optimally designed PPN contract (dashed line). For comparison, we also include the issuer value with straight debt (thin black line), which coincides with $\pi = 0$.

Figure 9: Issuer Value and Default Probability with Optimally Designed PPN Contracts



The graph on the left shows the optimal issuer value. The graph on the right depicts the probability of default given the optimal leverage. The plots show the values for an issuer financed with straight debt (thin solid line), for an issuer financed with the standard PPN contract (thick gray line) and for an issuer financed with the optimally designed PPN contract (dashed line). We compute the values using the parameters $A_0 = 100$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, $\alpha = 0.25$, and $X_p = 100$.

We examined the hump-shaped curve for PPN financing with $\pi = 0.5$ earlier in Section 6.1. The issuer can always increase the value by adjusting the participation rate. Both curves agree for $\delta = 0.4$, where the optimal participation rate is approximately $\pi^* = 0.5$. For values below $\delta = 0.4$, the issuer can increase the value by lowering the participation rate. For values above $\delta = 0.4$, the issuer is better off by increasing the participation rate. Since the optimal participation rate is $\pi^* = 0$ for the low-risk issuer with $\delta = 0$, the corresponding issuer value agrees with the case of straight debt financing. The issuer value under the optimal participation rate declines with the risk weight δ up to values of $\delta = 0.9$ and then increases slightly.

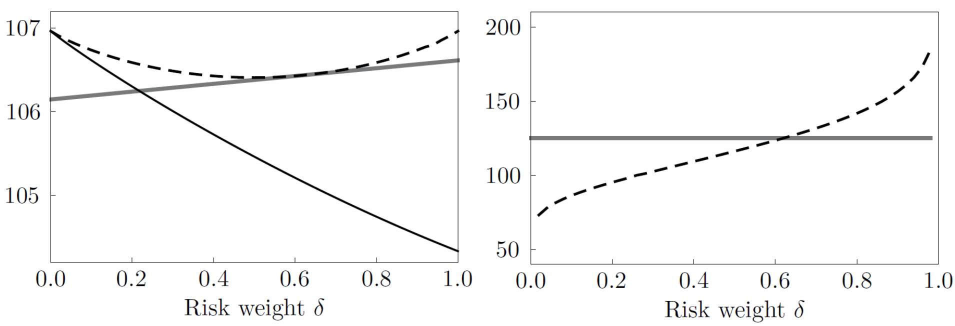
The optimization over the participation rate also has important consequences on the risk profile of the issuer. For low risk weights, the default probability is close to that of an issuer financed with straight debt. The probability of default monotonically increases with the risk weight δ, whereby it is always larger compared to an issuer financed purely with straight debt. For values of $\delta > 0.74$, the default probability of the optimally designed PPN contract surpasses that of the standard PPN contract with $\pi = 0.5$. The default probability of a high-risk issuer with $\delta = 1$ jumps from 5.1% up to 6.7%. This value is twice as high as the corresponding default probability of 3.3% with straight debt financing.

We conclude that allowing for optimal choice of the product parameters confirms our verdict on PPN financing by high-risk issuers. The issuer value can be increased at the expense of a considerably greater probability of default.

7.2 Discount Notes

For DCN financing, the product parameter of choice is the maximum repayment X_D. The issuer simultaneously determines the optimal volume $\bar{D} \geq 0$. We present the results of the optimization in *Figure 10*. The issuer value is shown on the left. The optimal maximum repayment amount X_D^* is plotted on the right. Both plots present the values for an issuer financed with the standard DCN contract with X_D = 125 (thick gray line) and the values for the optimally designed DCN contract (dashed line).

Figure 10: Issuer Value and Default Probability with Optimally Designed DCN Contracts



The graph on the left shows the optimal issuer value. The graph on the right depicts the repayment amount X_D. The plots show the values for an issuer financed with straight debt (thin solid line), for an issuer financed with the standard DCN contract (thick gray line) and for an issuer financed with the optimally designed DCN contract (dashed line). We compute the values using the parameters $A_0 = 100$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, and $\alpha = 0.25$.

Figure 10 shows that the cap X_D^* of the optimally designed DCN contract increases monotonically with δ. For δ = 0.62, the optimal cap is roughly equal to 125, which corresponds to the parameter of the standard DCN contract discussed in the previous sections. The maximum promised repayment is equal to 184 for the high risk issuer with δ = 1.

The most remarkable outcome is that the issuer who uses the optimally designed DCN contract never defaults for any given risk weight δ. So the favorable characteristic already derived in Section 6.2 is again observed. Consequently, the issuer value can be further increased.

7.3 Further Products

In this section, we test the robustness of our results for two different product types. As a representative for products with discontinuous payoffs, we consider *express notes* (ENs).

In addition, we analyze *short notes* (SNs), whose payoff decreases when the value of the underlying increases.

Formally, the promised payoff of an EN (see *Figure 14* in Appendix A.1) is given by

$$CE_T = \begin{cases} (1+r_E)\cdot \bar{E} & \text{if } \tilde{R}_T \geq X_E, \\ \min\left\{1, \frac{1}{X_L}\cdot \tilde{R}_T\right\}\cdot \bar{E} & \text{if } \tilde{R}_T < X_E. \end{cases} \tag{12}$$

If the value of the underlying asset at maturity is above the lower threshold X_L, then the investor receives the nominal amount $\bar{E}$. Should the underlying value end up above the upper threshold $X_E > X_L$, the investor receives an additional coupon payment of r_E. Should the underlying asset fall below X_L, the investor incurs a loss. Express notes can be thought of as discount notes with strike price X_L and an additional coupon payment above the second strike X_E, where the promised payoff has a jump.

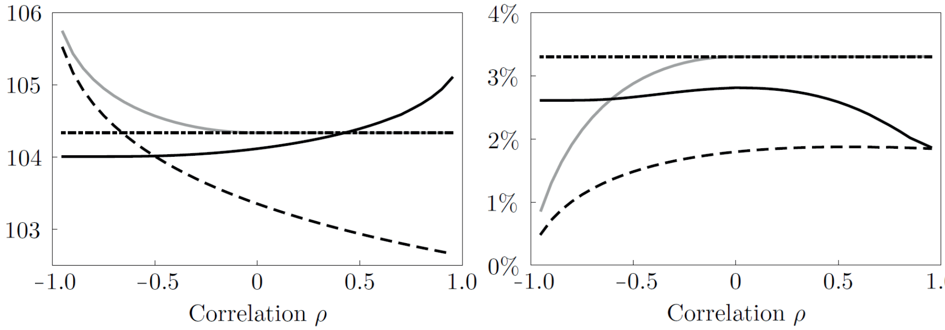
We represent the promised payoff of SNs (see *Figure 14* in Appendix A.1) as

$$CS_T = \max\left\{\frac{X_M - \tilde{R}_T}{X_M - X_S}, 0\right\}\cdot \bar{S}, \tag{13}$$

where the strike price X_S is usually set equal to the initial value R_0 of the underlying asset. The promised payoff of an SN is positive as long as $\tilde{R}_T < X_M$ with $X_M > X_S$. The investors get the maximum payoff $\frac{X_M}{X_M - X_S}\cdot \bar{S}$ when the value of the underlying drops to $\tilde{R}_T = 0$.

Figure 11 shows the issuer value and corresponding default probabilities depending on the correlation ρ for four different scenarios: straight debt financing (dot-dashed line), financing with ENs (solid black line), financing with SNs (dashed line) and financing with a mix of straight debt and SNs (solid gray line).

Figure 11: Issuer Value and Default Probabilities of Further Products



The plot on the left shows the optimal issuer value V_0 for straight debt financing (dot-dashed line), for EN financing (solid black line), for SN financing (dashed line) and for a financing mix of short notes and straight debt (solid gray line) depending on the correlation ρ. The plot on the right shows the corresponding risk-neutral default probabilities. We compute the values using the model parameters $A_0 = 100$, $\sigma_A = 0.2$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, and $\alpha = 0.25$ and product parameters $X_L = 50$, $X_E = 100$, $r_E = 30\%$, $X_S = 100$, and $X_M = 200$.

The value of an issuer financed with ENs increases monotonically with the correlation ρ. The graph looks similar to the issuer value with DCN financing (see *Figure 5*). Thus, the findings from Section 5 are again confirmed. For high positive correlations, the issuer can increase its value by financing with ENs. The default probability can be reduced for any correlation. In addition, an issuer financing with a mix of straight debt and ENs (not shown) always adds a positive fraction of ENs to the financing mix for all positive correlations.

The results for SN financing reverse the results from Section 5. Due to the negative relation between the SN payoff and the underlying, the issuer benefits from SNs when the correlation between the asset value return and the underlying asset's return is negative. The issuer value with SN financing decreases with the correlation. Issuers add SNs to a financing mix with straight debt for all negative correlations. In addition, the default probability can be significantly reduced for all correlations. Hence, SNs possess an insurance property similar to that of DCNs. We conclude that our results are robust to important variations on the payoff of RSPs.

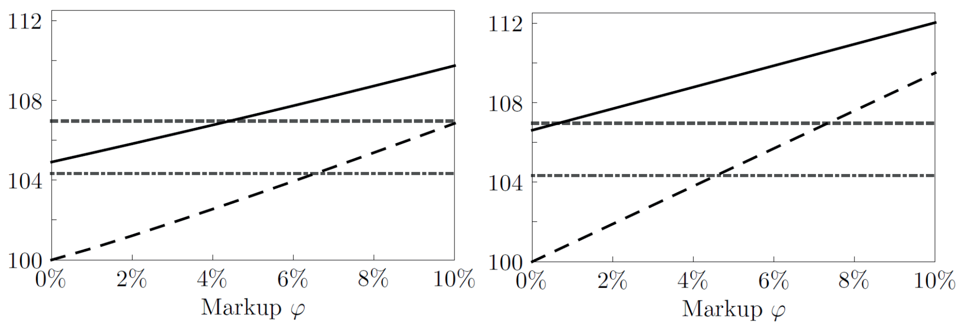
7.4 Product Complexity

An important empirical observation is that issuers sell RSPs to retail investors at a sizable markup. Stoimenov and Wilkens (2005) report an average markup at issuance of 3.9% for the German market. This markup increases with the complexity of the products. In a theoretical contribution, Carlin (2009) establishes a link between product complexity and the ability to generate profits from that particular product.

In this section, we test the robustness of our model with respect to this empirical observation. We incorporate the additional friction that the issuer is able to sell the RSP at a markup φ on the fair value. Such a markup comprises fees for sales, structuring, and depository. The markup φ is an upfront fee that investors have to pay at issuance. Hence, the fair value materializes directly after issuance and the market remains free of arbitrage opportunities. The markup directly increases the size of the special dividend to equity holders, which is equal to $(1 + \varphi) \cdot D_0$.

Figure 12 shows the optimal issuer value (solid line) for PPN financing on the left and DCN financing on the right. We display a high-risk issuer with $\delta = 1$, since RSPs are used to the maximum extent by this issuer. For comparison, *Figure 12* also shows the issuer value under straight debt financing for a low-risk issuer with $\delta = 0$ (dashed horizontal line) and for a high-risk issuer with $\delta = 1$ (dot-dashed horizontal line). Obviously, both are independent of the product markup φ.

Figure 12: Issuer Value with Product Markup



The plot on the left shows the optimal issuer value with PPN financing (solid line) depending on the product markup φ. The plot on the right shows the optimal issuer value with DCN financing (solid line). Both plots include the optimal issuer value with straight debt financing for a high-risk issuer with $\delta = 1$ (dot-dashed horizontal line) and a low-risk issuer with $\delta = 0$ (dashed horizontal line). Both plots also include the issuer value for PPN financing and DCN financing for a scenario without tax benefits (dashed line). We compute the values using the parameters $A_0 = 100$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, $\alpha = 0.25$, $X_P = 100$, $\pi = 0.5$, and $X_D = 125$.

The optimal value of an issuer financed with RSPs nearly linearly increases with the product markup φ. For both product types, the issuer value is greater compared to the high-risk issuer with straight debt financing. For PPNs, a product markup of $\varphi > 4.4\%$ is required such that PPN financing with $\delta = 1$ is advantageous to straight debt financing when $\delta = 0$. Hence, the maximum value from Section 6 is exceeded. For DCNs, a very low markup of at least $\varphi > 0.7\%$ is required such that DCN financing is beneficial.

We regard the product markup φ as a substitute for the tax benefit of debt. *Figure 12* also shows the respective issuer value for a scenario without tax benefits (dashed line), i.e., $\tau = 0$. For both product types, the issuer value is nearly linearly increasing in the markup φ.

We conclude that our results are robust to the specific implementation of the friction. However, the financing benefit must be linked to the outstanding volume of the RSP.

8 Conclusion

So far, the literature on retail structured products has focused on the profit maximizing behavior of the issuer. We contribute two new themes to this literature. First, we argue that RSPs are a valuable funding source for the issuer. Consequently, the investors in RSPs are to some extent exposed to the issuer's business risk. Second, we show that RSPs can be used for risk management purposes. The use of RSPs as a hedging instrument enables issuers to transfer risks outside the financial system. In this paper, we evaluate the conditions under which RSPs can indeed have a positive impact not only on the issuer value, but also on the default probability.

In the context of our model, we show that low-risk issuers still use straight debt financing, but high-risk issuers prefer RSP financing over straight debt. By holding the leverage ratio constant, high-risk issuers can increase the firm value and at the same time decrease the probability of default. Nevertheless, the issuer has an incentive to optimally adjust the leverage ratio and asset risk weight. Even when accounting for these optimal decisions, RSPs are added to the financing mix when the correlation between the issuer's assets and the underlying asset is positive and when the assets are risky. Issuers with high asset risk thereby increase the probability of default when issuing PPNs, but they reduce it by issuing DCNs. The results also hold when the issuer can optimally design the RSP. Furthermore, our results are also robust to the empirically observed friction of a markup on the RSP's fair value charged by the issuer.

Adding retail structured products to the financing mix is especially beneficial when the value of the issuer's assets strongly depends on the value of the underlying asset of the RSPs. The underlying asset can either be directly included in the asset portfolio as, for example, part of direct investments, or as inventories for trading. In addition, some other components of the asset portfolio might be highly correlated with the underlying asset. For example, the value of a loan provided by the bank is highly correlated with the value of the debtor's equity, since both claims can be thought of as claims contingent on the debtor's assets. Hence, we conclude that the issuer's asset portfolio can be decomposed into a component which depends on the underlying and residual component.

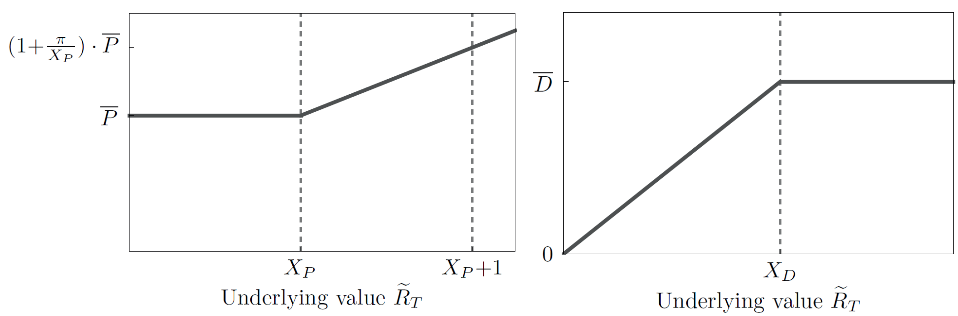
Our model centers around a hedging error caused by the mismatch between the payoffs of the issuer's assets and liabilities. Because of the high degree of customization of retail structured products, perfect hedges are often not feasible. In addition, hedging transactions only reduce the risk exposure of the aggregated financial sector if the counterpart sits outside the financial system. Retail investors are ideal counterparts of hedging transactions, since they arguably incur lower bankruptcy cost compared to financial institutions and because their small size limits contagion.

When a perfect hedge should indeed be feasible, the issuer can convert the liability from RSPs into a zero bond. As we show in our analysis, the issuer actually does not choose the perfect hedge when the asset portfolio is highly correlated to the underlying. In this case, RSPs offer advantageous features compared to straight debt. DCNs possess the property of a lower repayment when the issuer's asset value declines. PPNs generate a funding advantage over straight debt in return for sharing potential gains.

Appendix

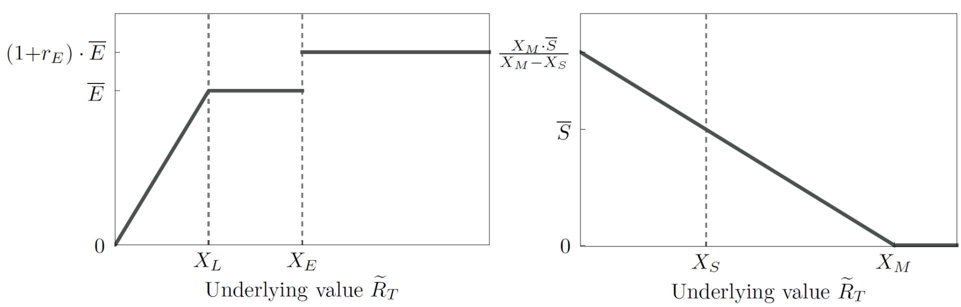
A.1 Payoffs of Retail Structured Products

Figure 13: Promised Payoff of Standard Products



The graph on the left shows the promised payoff CP_T of a principal-protected note with strike price $X_P = R_0$ and participation rate π. The graph on the right shows the promised payoff CD_T of a discount note with strike price X_D.

Figure 14: Promised Payoff of Further Products



The graph on the left shows the promised payoff CE_T of an express note with strike price X_E and coupon r_E. The investor incurs losses for values of the underlying below X_L. The graph on the right shows the promised payoff CS_T of a short note with strike price $X_S = R_0$ and upper cap X_M.

A.2 Valuation

A.2.1 Notation

The option pricing theory developed by Black and Scholes (1973) and Merton (1973) provides the framework for the pricing of the claims. To improve readability, we use the following short-hand notation throughout this section.

$$N_1(X) = N[d_1(X)], \tag{A.1}$$

$$N_{-1}(X) = N[-d_1(X)] = 1 - N_1(X), \tag{A.2}$$

$$N_2(X) = N[d_2(X)], \tag{A.3}$$

$$N_{-2}(X) = N[-d_2(X)] = 1 - N_2(X). \tag{A.4}$$

where X denotes the strike price and $N[y]$ denotes the standard normal cumulative distribution function. The terms d_1 and d_2 are defined as

$$d_1(X) = \frac{\ln\left(\frac{R_0}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \tag{A.5}$$

$$d_2(X) = \frac{\ln\left(\frac{R_0}{X}\right) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = d_1(X) - \sigma\sqrt{T}. \tag{A.6}$$

The values of European call options c_0 and put options P_0 with strike X are given by

$$c_0(X) = R_0 \cdot N_1(X) - X \cdot e^{-rT} \cdot N_2(X), \tag{A.7}$$

$$p_0(X) = X \cdot e^{-rT} \cdot N_{-2}(X) - R_0 \cdot N_{-1}(X). \tag{A.8}$$

A.2.2 Principal-Protected Notes

We first examine the case of financing with bonds and principal-protected notes. There are three conditions determining whether the issuer defaults. First, if the risk-free component of the asset portfolio does not suffice to repay the debt's principal, i.e., $\overline{B} + \overline{P} > (1 - \delta)F_T$, then the issuer defaults for small values of the underlying $R_T < X_1$ with

$$X_1 = \frac{\overline{B} + \overline{P} - (1-\delta)F_T}{\delta}. \tag{A.9}$$

This case obviously requires $\delta > 0$. For $\delta = 0$, the issuer defaults independent of the outcome of R_T.

Second, in case the issuer fails to settle the liability from the option embedded in the principal-protected note for some outcomes, i.e., if $\delta - \frac{\pi}{X_P} \cdot \overline{P} < 0$, it defaults for values of the underlying $R_T > X_2$ with

$$X_2 = \frac{\overline{B} + (1-\pi)\overline{P} - (1-\delta)F_T}{\delta - \frac{\pi}{X_P} \cdot \overline{P}}. \tag{A.10}$$

Third, the issuer might as well default for values of R_T below X_2. This situation happens when the principal amount is high, i.e., $\overline{B} + \overline{P} > (1 - \delta)F_T + \delta \cdot X_P$, and the participation rate is low with $\pi < \frac{\delta}{\overline{P}} \cdot X_P$.

The following table summarizes the resulting six possible scenarios. The second column shows for which realizations R_T of the risky asset the issuer defaults. Columns 3 to 5 show for which choice of parameters $\overline{B}$, $\overline{P}$, and δ the respective scenario occurs. The final column shows the risk-neutral probability of default of the issuer for each scenario.

Case i	Default	$\overline{B} + \overline{P}$	$\delta - \frac{\pi}{X_P} \cdot \overline{P}$	δ	pd
1	never	$\leq A_{0,T}$	≥ 0	≥ 0	0
2	$> X_2$	$\leq A_{0,T}$	< 0	≥ 0	$N_2(X_2)$
3	$< X_1$	$(> A_{0,T}) \wedge (\leq A_{X_P,T})$	≥ 0	> 0	$N_{-2}(X_1)$
4	$(< X_1) \wedge (> X_2)$	$(> A_{0,T}) \wedge (\leq A_{X_P,T})$	< 0	> 0	$N_2(X_2) + N_{-2}(X_1)$
5	$< X_2$	$> A_{X_P,T}$	> 0	> 0	$N_{-2}(X_2)$
6	always	$> A_{X_P,T}$	≤ 0	≥ 0	1

We denote the payoff of the assets for $R_T = 0$ as $A_{0,T} = (1-\delta)F_T$ and abbreviate the payoff of the assets for $R_T = X_P$ with $A_{X_P,T} = (1-\delta)\ F_T + \delta \cdot X_P$. In addition, we introduce the following short-hand notations:

$$g_1 = R_0 \cdot \left(N_1(X_P) - N_1(X_2)\right) - X_P \ e^{-rT} \cdot \left(N_2(X_P) - N_2(X_2)\right), \quad \text{(A.11)}$$

$$g_2 = \delta \cdot R_0 \cdot \left(N_1(X_2) - N_1(X_1)\right) + (1-\delta) \cdot F_0 \cdot \left(N_2(X_2) - N_2(X_1)\right), \quad \text{(A.12)}$$

$$g_3 = R_0 \cdot N_1(X_2) - X_P \cdot e^{-rT} \cdot N_2(X_2). \quad \text{(A.13)}$$

The total firm value of the issuer $V_0^i\left(\bar{B}, \bar{P}\right)$ for the respective case i is given by

$$V_0^1 = A_0 + \tau\left(1 - e^{-rT}\right)\left(\left(\bar{B} + \bar{P}\right)e^{-rT} + \frac{\pi}{X_P}\bar{P} \cdot c_0(X_P)\right), \quad \text{(A.14)}$$

$$\begin{aligned} V_0^2 = \ & A_0 + \tau\left(1 - e^{-rT} N_{-2}(X_2)\right)\left(\left(\bar{B} + \bar{P}\right)e^{-rT} N_{-2}(X_2) + \frac{\pi}{X_P}\bar{P} \cdot g_1\right) \\ & -\left(\alpha + (1-\alpha)\tau e^{-rT} N_{-2}(X_2)\right)\left(\delta R_0 N_1(X_2) + (1-\delta) F_0 N_2(X_2)\right), \end{aligned} \quad \text{(A.15)}$$

$$\begin{aligned} V_0^3 = \ & A_0 + \tau\left(1 - e^{-rT} N_2(X_1)\right)\left(\left(\bar{B} + \bar{P}\right)e^{-rT} N_2(X_1) + \frac{\pi}{X_P}\bar{P} \cdot c_0(X_P)\right) \\ & -\left(\alpha + (1-\alpha)\tau e^{-rT} N_2(X_1)\right)\left(\delta R_0 N_{-1}(X_1) + (1-\delta) F_0 N_{-2}(X_1)\right), \end{aligned} \quad \text{(A.16)}$$

$$\begin{aligned} V_0^4 = \ & A_0 + \tau\left(1 - e^{-rT} \cdot \left(N_2(X_1) - N_2(X_2)\right)\right) \\ & \cdot \left(\left(\bar{B} + \bar{P}\right)e^{-rT}\left(N_2(X_1) - N_2(X_2)\right) + \frac{\pi}{X_P} \cdot \bar{P} \cdot g_1\right) \\ & -\left(\alpha + (1-\alpha)\tau e^{-rT} \cdot \left(N_2(X_1) - N_2(X_2)\right)\right) \cdot g_2, \end{aligned} \quad \text{(A.17)}$$

$$\begin{aligned} V_0^5 = \ & A_0 + \tau\left(1 - e^{-rT} N_2(X_2)\right)\left(\left(\bar{B} + \bar{P}\right)e^{-rT} N_2(X_2) + \frac{\pi}{X_P}\bar{P} \cdot g_3\right) \\ & -\left(\alpha + (1-\alpha)\tau e^{-rT} N_2(X_2)\right)\left(\delta R_0 N_{-1}(X_2) + (1-\delta) F_0 N_{-2}(X_2)\right), \end{aligned} \quad \text{(A.18)}$$

$$V_0^6 = (1-\alpha) A_0. \quad \text{(A.19)}$$

A.2.3 Discount Notes

We examine the claim values for the issuer financed with bonds and discount notes. The issuer's payoff is characterized by two default thresholds. First, if the risk-free portion of the asset portfolio is exceeded by the minimum debt payment, i.e., $\bar{B}+\bar{D}>(1-\delta)F_T$, then the issuer defaults for small values of the underlying below the threshold $R_T < X_3$ with

$$X_3 = \frac{\bar{B}-(1-\delta)F_T}{\delta-\gamma\bar{D}}. \tag{A.20}$$

This case requires $\delta-\gamma\bar{D}>0$.

The default boundary X_3 is relevant for a further scenario. If the bank issues an amount of discount notes exceeding the value of its investment in the risky asset, i.e., $\delta-\gamma\bar{D}<0$, then the bank defaults for values above the threshold $R_T > X_3$.

Second, the issuer's liabilities are in any case limited to $\bar{B}+\bar{D}$. Thus, the issuer never defaults for values of the underlying $R_T > X_4$ with

$$X_4 = \frac{\bar{B}+\bar{D}-(1-\delta)F_T}{\delta}. \tag{A.21}$$

This case requires $\delta > 0$. Otherwise, the default boundaries become independent of R_T.

The following table summarizes the resulting 6 possible scenarios. The second column shows for which realizations R_T of the risky asset the issuer defaults. Columns 3 to 5 show for which choice of parameters $\bar{B}$, $\bar{D}$, and δ the respective scenario occurs. The final column shows the risk-neutral probability of default of the issuer for each scenario.

Case i	Default	$\bar{B}$	$\bar{B}+\bar{D}$	δ	pd
1	never	$\le A_{0,T}$	$\le A_{X_D,T}$	≥ 0	0
2	$(< X_4)\wedge(> X_3)$	$\le A_{0,T}$	$> A_{X_D,T}$	> 0	$N_2(X_4)+N_{-2}(X_3)$
3	$> X_3$	$\le A_{0,T}$	$> A_{X_D,T}$	$= 0$	$N_2(X_3)$
4	$< X_3$	$> A_{0,T}$	$\le A_{X_D,T}$	≥ 0	$N_{-2}(X_3)$
5	$< X_4$	$> A_{0,T}$	$> A_{X_D,T}$	> 0	$N_{-2}(X_4)$
6	always	$> A_{0,T}$	$> A_{X_D,T}$	$= 0$	1

We denote the payoff of the assets for $R_T = 0$ as $A_{0,T} = (1-\delta)F_T$ and abbreviate the payoff of the assets for $R_T = X_D$ with $A_{X_D,T} = (1-\delta)F_T + \delta \cdot X_D$. We introduce the following short-hand notations:

$$g_4 = \delta R_0 \left(N_1(X_3) - N_1(X_4)\right) + (1-\delta) F_0 \left(N_2(X_3) - N_2(X_4)\right), \tag{A.22}$$

$$g_5 = \delta R_0 N_{-1}(X_3) + (1-\delta) F_0 N_{-2}(X_3), \tag{A.23}$$

$$g_6 = \delta R_0 N_{-1}(X_4) + (1-\delta) F_0 N_{-2}(X_4) \tag{A.24}$$

$$g_7 = X_D e^{-rT} \left(N_2(X_3) - N_2(X_D)\right) - R_0 \left(N_1(X_3) - N_1(X_D)\right). \tag{A.25}$$

The total firm value of the issuer $V_0^i(\bar{B}, \bar{D})$ for the respective case i is given by

$$V_0^1 = A_0 + \tau\left(1 - e^{-rT}\right)\left(\left(\bar{B} + \bar{D}\right)e^{-rT} - \gamma \bar{D} \cdot p_0(X_D)\right), \tag{A.26}$$

$$\begin{aligned} V_0^2 = \; & A_0 + \tau\left(1 - e^{-rT}\left(N_2(X_4) - N_2(X_3)\right)\right) \\ & \cdot \left(\left(\bar{B} + \bar{D}\right)e^{-rT}\left(N_2(X_4) - N_2(X_3)\right) - \gamma \bar{D} \cdot g_7\right) \\ & -\left(\alpha + (1-\alpha)\tau e^{-rT}\left(N_2(X_4) - N_2(X_3)\right)\right) \cdot g_4, \end{aligned} \tag{A.27}$$

$$\begin{aligned} V_0^3 = \; & A_0 + \tau\left(1 - e^{-rT} N_{-2}(X_3)\right)\left(\bar{B} e^{-rT} N_{-2}(X_3) + \gamma \bar{D} \cdot R_0 N_{-1}(X_3)\right) \\ & -\left(\alpha + (1-\alpha)\tau e^{-rT} N_{-2}(X_3)\right) \cdot F_0 N_2(X_3), \end{aligned} \tag{A.28}$$

$$\begin{aligned} V_0^4 = \; & A_0 + \tau\left(1 - e^{-rT} N_2(X_3)\right)\left(\left(\bar{B} + \bar{D}\right)e^{-rT} N_2(X_3) - \gamma \bar{D} \cdot g_7\right) \\ & -\left(\alpha + (1-\alpha)\tau e^{-rT} N_2(X_3)\right) \cdot g_5, \end{aligned} \tag{A.29}$$

$$\begin{aligned} V_0^5 = \; & A_0 + \tau\left(1 - e^{-rT} N_2(X_4)\right)\left(\left(\bar{B} + \bar{D}\right)e^{-rT} N_2(X_4)\right) \\ & -\left(\alpha + (1-\alpha)\tau e^{-rT} N_2(X_4)\right) \cdot g_6, \end{aligned} \tag{A.30}$$

$$V_0^6 = (1-\alpha) A_0. \tag{A.31}$$

A.3 Proofs of Propositions

A.3.1 High-Risk Issuer and PPN Financing

We restrict the participation rate to the typical case of $\pi \leq 1$. Since we would like to compare PPN financing to straight debt financing, we consider points where the issuers have equal probability of default under both financing choices, i.e. $pd_B = pd_P$. Under this condition, we can express the issuer value $V_{P,0}$ and the value of the principal-protected note CP_0 as

$$CP_0\left(\overline{P}\right) = B_0\left(\overline{B} = X\right) + a, \tag{A.32}$$

$$V_{P,0}\left(\overline{P}\right) = V_{B,0}\left(\overline{B} = X\right) + a \cdot b, \tag{A.33}$$

with

$$X = \begin{cases} \overline{P} & \text{for } 0 < \overline{P} \leq X_P\,, \\ \dfrac{(1-\pi)\overline{P}}{1 - \dfrac{\pi}{X_P} \cdot \overline{P}} & \text{for } X_P < \overline{P} \leq \dfrac{X_P}{\pi}, \end{cases} \tag{A.34}$$

$$a = \begin{cases} \dfrac{\pi}{X_P} \cdot \overline{P} \cdot c_0\left(X_P\right) & \text{for } 0 < \overline{P} \leq X_P\,, \\ \dfrac{\pi}{X_P} \cdot \overline{P} \cdot \left(R_0 N_1\left(X\right) - X_P \cdot e^{-rT} N_2\left(X\right)\right) & \text{for } X_P < \overline{P} \leq \dfrac{X_P}{\pi}, \end{cases} \tag{A.35}$$

$$b = \tau\left(1 - e^{-rT} N_2\left(X\right)\right) \tag{A.36}$$

We want to show for a given probability of default at $\overline{B} = X$ and $\pi > 0$ that

$$\begin{aligned} & \lambda_B < \lambda_P \\ \Leftrightarrow\quad & \frac{B_0}{V_{B,0}} < \frac{CP_0}{V_{P,0}} \\ \Leftrightarrow\quad & \frac{B_0}{V_{B,0}} < \frac{B_0 + a}{V_{B,0} + a \cdot b} \\ \Leftrightarrow\quad & b \cdot B_0 < V_{B,0} \end{aligned} \tag{A.37}$$

The last relation is always true, since $b < 1$ and by definition $B_0 \leq V_{B,0}$. This result directly proves proposition 1 for PPN financing.

Moreover, this result also proves proposition 2 for PPN financing. For each value of $\bar{B}$, we can find a corresponding point for PPN financing with equal probability of default, which produces a higher firm value $V_{P,0} = V_{B,0} + a \cdot b > V_{B,0}$ and also a higher leverage ratio $\lambda_P > \lambda_B$. Since this finding is true for any value of $\bar{B}$, the graph of $V_{P,0}$ has to be strictly above the $V_{B,0}$ graph for any attainable leverage ratio $\lambda < 1$.

Another way of showing this result is using the first derivatives of the issuer value $V_{P,0}$ and the leverage ratio λ_P with respect to the participation rate π. We note that straight debt financing can be represented by $\pi = 0$.

$$\frac{\partial V_{P,0}}{\partial \pi} = \frac{a \cdot b}{\pi} > 0, \tag{A.38}$$

$$\frac{\partial \lambda_P}{\partial \pi} = \frac{a}{\pi \cdot (V_{B,0} + a \cdot b)^2} \cdot \left(V_{B,0} - b \cdot B_0\right) > 0. \tag{A.39}$$

Both derivatives are always positive. Hence, as long as $\bar{P} \leq \frac{X_P}{\pi}$, an increase in the participation rate always creates value, but does not increase the bankruptcy cost, since $\frac{\partial pd_P}{\partial \pi} = 0$. PPN financing is strictly superior to straight debt financing.

The case of $\bar{P} > \frac{X_P}{\pi}$ is not of interest, since the issuer always defaults, i.e., $\lambda_P = 1$ and $V_{P,0} = CP_0 = (1 - \alpha) R_0$. The leverage ratio of $\lambda_B = 1$ cannot be attained under straight debt financing. The above outlined proof holds analogously for PPN designs with $\pi > 1$ as long as $\bar{P} \leq \frac{X_P}{\pi}$.

A.3.2 High-Risk Issuer and DCN Financing

The relevant risk-neutral default probabilities are given by

$$pd_B\left(\bar{B}\right) > 0, \tag{A.40}$$

$$pd_D\left(\bar{D}\right) = \begin{cases} 0 & \text{for } \bar{D} \leq X_D, \\ pd_B\left(\bar{B} = \bar{D}\right) & \text{for } \bar{D} > X_D. \end{cases} \tag{A.41}$$

In the case of $\bar{B} > X_D$, the debt claim values are also identical, i.e., $B_0\left(\bar{B}\right) = CD_0\left(\bar{D} = \bar{B}\right)$. The issuer values and leverage ratios agree as well. Hence, the default probability with

DCN financing is either zero or agrees with the corresponding probability under straight debt financing. This finding proves Proposition 1 for discount notes.

This reasoning also proves proposition 2 for $\bar{D} > X_D$. In the remaining case of $\bar{D} > X_D$, we express the issuer value under DCN financing as

$$V_{0,D}(\lambda) = R_0 \cdot \frac{1}{1-\left(1-e^{-rT}\right)\tau\lambda}. \tag{A.42}$$

An issuer financed only with straight debt defaults for realizations of the underlying asset $R_T < \bar{B}$. The resulting issuer value is given by

$$V_{0,B}(\lambda) = R_0 \cdot \frac{1-\alpha' \ N_{-1}\left(\bar{B}\right)}{1-\left(1-e^{-rT}\left(1-N_{-2}\left(\bar{B}\right)\right)\right)\tau\lambda} \tag{A.43}$$

with $\alpha' = \alpha + \tau(1-\alpha)$ and $\tau \leq \alpha' \leq 1$. The term $N_{-2}\left(\bar{B}\right)$ corresponds to the risk-neutral default probability pd_B. To simplify the expression, we use the relation $N_{-1}\left(\bar{B}\right) = N_{-2}\left(\bar{B}\right) - \varepsilon$ with $0 \leq \varepsilon \leq 1$. The issuer value now reads

$$V_{0,B}(\lambda) = R_0 \cdot \frac{1-\alpha' \ pd_B + \alpha'\varepsilon}{1-\left(1-e^{-rT}\right)\tau\lambda - pd_B e^{-rT}\tau\lambda}. \tag{A.44}$$

For $pd_B = 0$, which also implies $\varepsilon = 0$, the issuer value $V_{0,B}$ under straight debt financing agrees with the issuer value $V_{0,D}$ under DCN financing. We inspect the derivative of the issuer value with respect to the default probability given by

$$\frac{\partial V_{0,B}(\lambda)}{\partial pd_B} = \frac{R_0}{(\ldots)^2} \cdot \left(-\alpha' + \left(\alpha' + e^{-rT}\left(1-\alpha'\right)\right)\tau\lambda + \alpha' e^{-rT}\tau\lambda\,\varepsilon\right). \tag{A.45}$$

The term in brackets is negative for $\varepsilon = 0$. Proposition 2 requires this derivative to be negative. Hence, we need to impose a condition of the form

$$\varepsilon \leq \bar{\varepsilon} = \frac{\alpha' - \left(\alpha' + e^{-rT}\left(1-\alpha'\right)\right)\tau\lambda}{\alpha' \ e^{-rT}\tau\lambda}. \tag{A.46}$$

Both numerator and denominator are positive and smaller than 1. The upper boundary $\bar{\varepsilon}$ increases with the bankruptcy cost α and decreases with the leverage ratio λ and the tax rate τ. This restriction puts an upper boundary on $\sigma\sqrt{T}$, since $N(d_1) = N(d_2 + \sigma\sqrt{T})$. The restriction is not binding for typical parameter choices.

A.3.3 Low-Risk Issuer and DCN Financing

The risk-neutral probability of default of the issuer financed with bonds is $pd_B = 0$ for all attainable leverage ratios $\lambda < 1$. This directly proves Proposition 3 since the default probability of an issuer financed with DCN is positive at least for some $\lambda < 1$.

The issuer value under straight debt financing is given by

$$V_{0,B}(\lambda) = F_0 \cdot \frac{1}{1-\left(1-e^{-rT}\right)\tau\lambda}. \tag{A.47}$$

The maximum attainable leverage ratio without default is at $\bar{B} = F_0 \cdot e^{rT}$ with

$$\lambda_B^{\max} = \frac{1}{1+\left(1-e^{-rT}\right)\tau}. \tag{A.48}$$

We need to consider two cases. In the first case with $\bar{D} \leq F_0 \cdot e^{rT}$, the issuer is not defaulting. The issuer value is given by

$$V_{0,D}(\lambda) = F_0 \cdot \frac{1}{1-\left(1-e^{-rT}\right)\tau\lambda}, \tag{A.49}$$

which agrees with the issuer value $V_{0,B}$ under straight debt financing.

The maximum attainable leverage ratio without default is at $\bar{D} = F_0 \cdot e^{rT}$ with

$$\lambda_D^{\max} = \frac{1}{q+\left(1-e^{-rT}\right)\tau}. \tag{A.50}$$

with $q = \dfrac{e^{-rT}}{e^{-rT} - \gamma p_0(X_D)}$. From $q > 1$ follows that $\lambda_D^{\max} < \lambda_B^{\max}$.

In the second case with $\bar{D} > F_0 \cdot e^{rT}$, the issuer defaults for realizations of the underlying above the threshold $X_3 = \frac{F_0 \cdot e^{rT}}{\gamma \bar{D}}$. The issuer value is given by

$$V_{0,D}(\lambda) = F_0 \cdot \frac{1 - pd_D \cdot \tau - \alpha\left(1 - \left(\tau + (1-\tau)(1 - pd_D)\right)\right)}{1 - \left(1 - e^{rT}(1 - pd_D)\right)\tau\lambda}. \tag{A.51}$$

Since an increase in the bankruptcy cost α always leads to a decrease in the issuer value, i.e., $\frac{\partial V_{0,D}}{\partial \alpha} < 0$, we can consider the limiting case of $\alpha = 0$. The resulting claim value is given by

$$V_{0,D}(\lambda)\Big|_{\alpha=0} = F_0 \cdot \frac{1 - pd_D \cdot \tau}{1 - \left(1 - e^{rT}\right)\tau\lambda - pd_D \cdot e^{-rT}\tau\lambda}. \tag{A.52}$$

For $pd_D = 0$, the issuer value agrees with the value $V_{0,B}$ under straight debt financing. We inspect the first derivative with respect to the default probability given by

$$\frac{\partial V_{0,D}(\lambda)}{\partial pd_D}\Bigg|_{\alpha=0} = -F_0 \cdot \frac{\tau\left(1 - \lambda\left(e^{-rT} + \tau\left(1 - e^{-rT}\right)\right)\right)}{(\ldots)^2}. \tag{A.53}$$

The numerator is always positive. Hence, an increase in the default probability always results in a loss of value even for the limiting case of $\alpha = 0$. This proves Proposition 4 for DCN financing.

A.3.4 Low-Risk Issuer and PPN Financing

The risk-neutral probability of default of the issuer financed with bonds is $pd_B = 0$ for all attainable leverage ratios $\lambda < 1$. This relation directly proves Proposition 3, since the default probability of an issuer financed with PPN is positive at least for some $\lambda < 1$.

We need to consider two cases. In the first case with $\bar{P} > F_T = F_0 \cdot e^{rT}$, the issuer always defaults. For the resulting leverage ratio of $\lambda < 1$, the issuer value is independent of the financing choice.

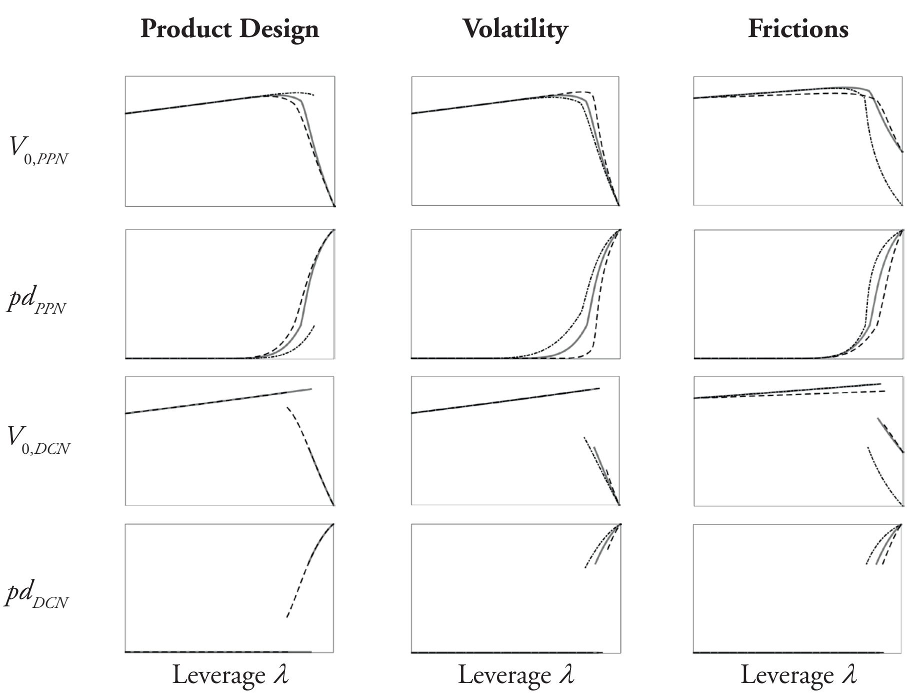
In the second case of $\bar{P} \leq F_T$, the issuer defaults for realizations of the underlying above the threshold $X_2 = \frac{X_T}{\pi \cdot \bar{P}}\left(F_T - (1-\pi)\bar{P}\right)$. The corresponding claim value is given by

$$V_{0,P}(\lambda) = F_0 \cdot \frac{1 - pd_P \cdot \tau - \alpha\left(1 - \left(\tau + (1-\tau)(1 - pd_P)\right)\right)}{1 - \left(1 - e^{rT}(1 - pd_P)\right)\tau\lambda}. \qquad \text{(A.54)}$$

The functional form is the same as for the issuer value under DCN financing from Equation 51. Of course, the claim values are not the same, since pd_P and pd_D depend differently on the leverage ratio λ, but the above outlined proof for DCN financing with $\bar{D} > F_T$ is valid for all positive default probabilities. Hence, the same reasoning can be applied to prove Proposition 4 for PPN financing.

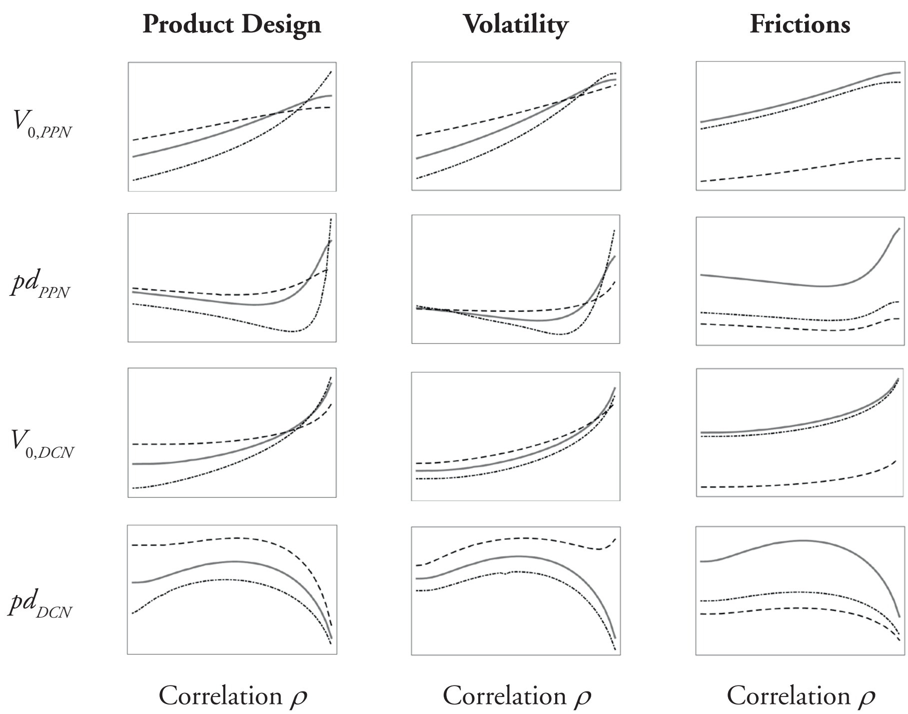
A.4 Comparative Static Analysis

Figure 15: Comparative Static Analysis for Constant Leverage



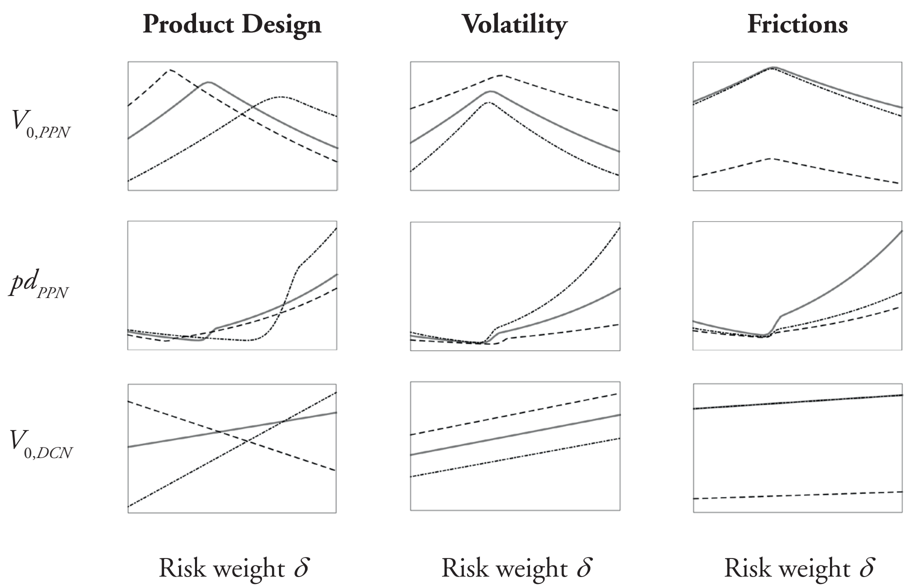
The graphs show the issuer value V_0 and the corresponding probability of default *pd* of a high risk issuer (see Section 4.1). We compute the base case (thick solid line) using the values $A_0 = 100$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, $\alpha = 0.25$ and product parameters $X_P = 100$, $\pi = 0.5$ and $X_D = 125$. The first column shows different product designs for the PPN with $\pi = 0.25$ (dashed line) and $\pi = 1$ (dot-dashed line) as well as for the DCN with $X_D = 100$ (dashed line). The second column shows two alternative scenarios for the volatility of the underlying with $\sigma_R = 0.1$ (dashed line) and $\sigma_R = 0.3$ (dot-dashed line). The third column shows two alternative scenarios for the frictions with a tax rate of $\tau = 0.25$ (dashed line) and bankruptcy costs of $\alpha = 0.5$ (dot-dashed line).

Figure 16: Comparative Static Analysis for Optimal Financing



The graphs show the issuer value V_0 and the corresponding probability of default *pd*. We compute the base case (thick solid line) using the values $A_0 = 100$, $\sigma_A = 0.2$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, $\alpha = 0.25$ and product parameters $X_P = 100$, $\pi = 0.5$ and $X_D = 125$. The first column shows different product designs for the PPN with $\pi = 0.25$ (dashed line) and $\pi = 1$ (dot-dashed line) as well as for the DCN with $X_D = 100$ (dashed line) and $X_D = 150$ (dot-dashed line). The second column shows two alternative scenarios for the volatility of the underlying with $\sigma_R = 0.1$ (dashed line) and $\sigma_R = 0.3$ (dot-dashed line). The third column shows two alternative scenarios for the frictions with a tax rate of $\tau = 0.25$ (dashed line) and bankruptcy costs of $\alpha = 0.5$ (dot-dashed line).

Figure 17: Comparative Static Analysis for Optimal Risk-Taking



The graphs show the issuer value V_0 and the corresponding probability of default *pd*. We compute the base case (thick solid line) using the values $A_0 = 100$, $\sigma_R = 0.2$, $r = 0.15$, $T = 1$, $\tau = 0.5$, $\alpha = 0.25$ and product parameters $X_P = 100$, $\pi = 0.5$, and $X_D = 125$. The first column shows different product designs for the PPN with $\pi = 0.25$ (dashed line) and $\pi = 1$ (dot-dashed line) as well as for the DCN with $X_D = 100$ (dashed line) and $X_D = 150$ (dot-dashed line). The second column shows two alternative scenarios for the volatility of the underlying with $\sigma_R = 0.1$ (dashed line) and $\sigma_R = 0.3$ (dot-dashed line). The third column shows two alternative scenarios for the frictions with a tax rate of $\tau = 0.25$ (dashed line) and bankruptcy costs of $\alpha = 0.5$ (dot-dashed line).

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